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PROBABILITY OF FAILURE CALCULATIONS FOR THERMAL PROTECTION TILES ON THE SPACE SHUTTLE "COLUMBIA"

by

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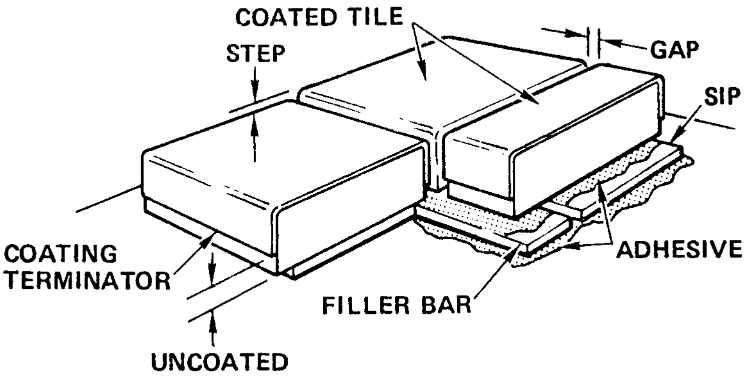
ABSTRACT

Following the loss of some tiles on one of the space shuttle vehicles on its way to Cape Kennedy in Florida and prior to the launch of the space shuttle "Columbia" on its maiden voyage on April 12, 1981, a number of independent investigations were made of the safety of the thermal protection system for expected thermal and aerodynamic loadings. One of these was carried out by the Brittle Materials Design Group (BMDG) of the University of Washington and is the basis of this paper.

A main component of the approach followed by the BMDG was a statistical prediction of the probability of failure (p.o.f.) of individual tiles under maximum expected environmental loadings for that tile. The strength of the silicon tile material is characterized by the parameters of the Weibull distribution function based on tests of specimens of the material and actual data from loading tests of individual tiles as mounted on the shuttle. These parameters were used in a Finite Element Program in which methods for predicting the probability of failure have been incorporated. This program and procedure was verified by application to panel tests on shuttle subcomponents using these values. The results were found consistent with observed results. The program was then used for prediction of probability of failure of critical tiles on the "Columbia" prior to the launch using expected maximum force and moment vectors and substrate deflections for individual standard tiles. Other parameter studies were also made and are included in the paper.

1. Introduction

The thermal protection system of the space shuttle "Columbia" includes over 30,000 insulation tiles. These tile are subjected to varying thermal and mechanical loadings during launch, orbit and re-entry depending upon location of the tile. The undersurface black tile are subjected to the highest temperatures and are essential for maintaining the strength of the aluminum structure during re-entry. Standard tiles are 6 inches square and about 2 inches thick made from pure fine silica fibers in a very porous brittle network. These are covered with a protective black glass coating forming a light weight reusable surface insulation. Because of the brittle characteristic of the tile, they must be isolated from the surface deformation of the shuttle structural system. This is achieved by strain isolation pads (SIP) made of nylon fiber and bonded between the skin of the shuttle and the tile by R.T.V. adhesive. Figure 1 shows the general configuration of the thermal protection system.



MATERIALS

TILE – 352 kg/m^3 (22 lb/ft^3) / 144 kg/m^3 (9 lb/ft^3) PURE SILICA FIBER–FIRED AT ~1370°C (~2500°F)

COATING – BOROSILICATE (GLASS) FOR WATERPROOF & THERMAL PROPERTIES – FIRED AT 1150–1205°C (2100-2200°F)

SIP – NOMEX FELT

FILLER – COATED NOMEX FELT

ADHESIVE – RTV SILICONE

Fig. 1. Silica Tile System Configuration

Because of the large number of tile, the critical dependency on the thermal protection, the uncertainties in loading and material properties, and the brittle characteristics of the tile, the authors proposed an independent statistical strength analysis of the tile of the thermal protection system combining Finite Element Analysis with a Weibull statistical characterization of the material properties of the tile. This analysis is described in the following sections with application to critical tile on the "Columbia" for anticipated loadings for its maiden flight.

2. Characterization of Materials

The material characterization was complicated by the non-homogeneous and non-isotropic nature of the tile material as used in the shuttle. The non-homogeneous character of the tiles comes from the "densification" coating put on the bonding surface of many of the tiles to increase the bond strength. The density and strength of this coating decrease with depth of penetration from the surface. In addition, the silica tiles have a definite through-the-thickness (TTT) orthotropy, Fig. 2, that was considered in the material characterization and stress and failure analysis. Two different tile materials, LI-900 and LI-2200, having densities of 9 lbs/ft^3 and 22 lbs/ft^3, respectively, are used on the shuttle.

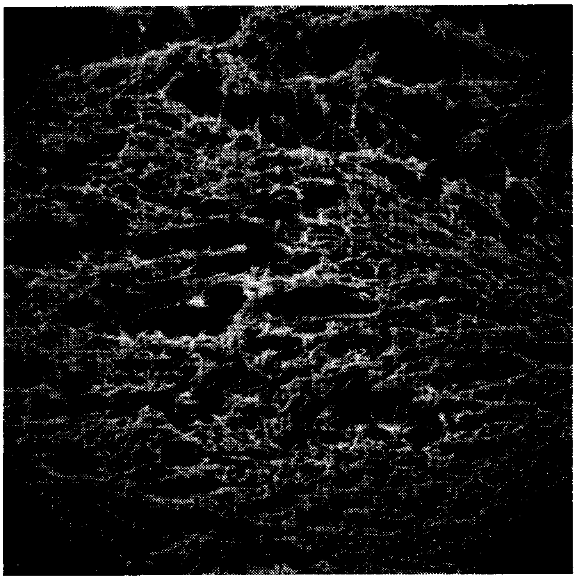


Fig. 2.
SEM photo LI-900
160 x

The thermal tiles are bonded to the surface of the Shuttle through the strain isolation pads (SIP) of smaller dimensions than the area of the tile surface. The SIP material is a woven nylon fiber material, Fig. 3, with hardening stress-strain characteristics under stress. This was modelled using a bi-linear stress-strain relationship (Fig. 4).

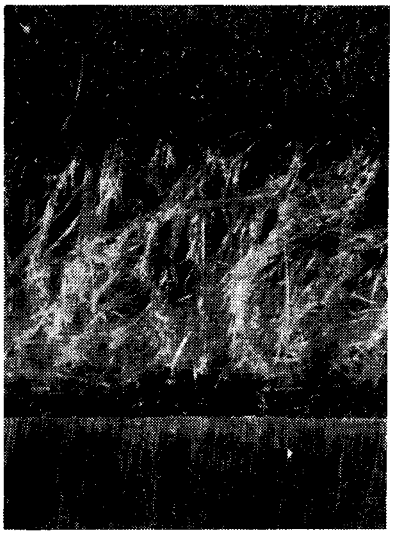


Fig. 3. AL/SIP/Tile System 11.5x

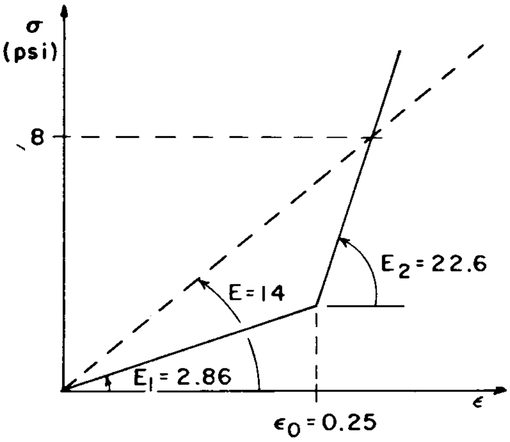


Fig. 4. Stress-Strain for SIP

A two parameter Weibull model[1,2] is used to characterize the strength of the silica tile material. Weibull parameters were determined for three separate layers in the densified layer of each LI-900 and LI-2200 tile and in the undensified material. These were also determined in the in-plane, i.e., parallel to the SIP, direction and in the TTT direction for each of the materials. The Weibull values were derived from a statistical analysis of the fracture stresses of rectangular bars tested in 4-point loading. Additional data was available from tests by NASA and Rockwell-International. The material parameters are summarized in Table 1.

3. Probability of Failure

A Weibull model of failure[1,2] is based on the assumtion that flaws are distributed at random with a certain density per unit volume. The

Table 1. Assumed Properties of LI-900, Densified Layers and SIP for FEM Results

	SIP	Undensified LI-900	Boundary Layers of Undensified LI-900	Lower Densified Layer LI-900	Mid-Densified Layer LI-900	Inner Densified Layer LI-900	Glaze Coating
E_x^* (psi)	600	28,000	28,000	145,000	135,000	120,000	$8x10^6$
E_y (psi)	14	7,000	7,000	17,500	16,500	15,000	-
ν	.4	.2	.2	.2	.2	.2	-
G_{xy} (psi)	214	3,000	3,000	13,890	13,070	11,840	-
σ_x^0	-	120	60	120	120	120	-
σ_y^0	- -	12	12	24	24	24	-
λ	-	6,10	6,10	6,10	6,10	6,10	-
Layer Thickness	-	-	.03125"	.03125"	.03125"	.06250"	11 mils

* x is the in-plane direction

y is the through-the-thickness direction

failure is based on the "weakest link hypothesis", which states that a component will fail when the stress intensity at any flaw reaches a critical value for crack propagation. Thus the structural component is represented as a series model or a chain, with components being small parts of the structure, in which the failure depends on the weakest component.

For the expression of the probability of failure a 2 parameter Weibull distribution is used, and the *probability of failure* of a small volume dV under a normal tensile stress σ is computed as:

$$P_f\ (dV) = 1\ -\exp\left[\ -\ \left(\frac{\sigma}{\sigma_0}\right)^m dV\right] \qquad (1)$$

where:

σ_0 and m are material constants, estimated from experimental data shown in references [3,4].

The *probability of survival* of a volume dV is expressed as:

$$P_s\,(dV) = \exp\left[-\left(\frac{\sigma}{\sigma_o}\right)^m dV\right] \tag{2}$$

and the *risk of rupture*

$$S(dV) = \left(\frac{\sigma}{\sigma_o}\right)^m dV \tag{3}$$

The risk of rupture of a volume V is

$$S(V) = \int_V \left(\int_{\Delta V} \frac{1}{\Delta V} \left(\frac{\sigma_n}{\sigma_o} \right)^m dV \right) \Delta V \tag{4}$$

where:

The first integral is over a small volume ΔV around a point, and σ_n is the average tensile stress for the small region dV at this point.

The second integral is over the volume of the structural component.

For the computation of the *probability of failure* in a finite element model,

$$S(V) = \sum_{k=1}^{N_e} \int_{V_k} \left(\int_{\Delta V} \frac{1}{\Delta V} \left(\frac{\sigma_n}{\sigma_o}\right)^m dV \right) \Delta V \tag{5}$$

where:

N_e is the number of elements.

For each element the stresses are computed at the Gauss quadrature points, so the integral over the volume of each element will be evaluated with a Gauss numerical integration scheme as [4].

$$\int_{V_k} \left(\int_{\Delta V} \frac{1}{\Delta V} \left(\frac{\sigma_n}{\sigma_o}\right)^m dV \right) \Delta V = \underbrace{V_k}_{\text{Element volume}} \sum_{i=1}^{M} \underbrace{W_i}_{\text{Integration weights}} \underbrace{\left(\int_{\Delta V} \frac{1}{\Delta V} \left(\frac{\sigma_n}{\sigma_o}\right)^m dV \right)_i}_{\text{Evaluated at Gauss quadrature points}} \tag{6}$$

Because of the orthotropy of the tile material, lack of material data for a better model for probability of failure in the orthotropic tile materials, and time constraints, Eq. (6) was reduced to integration

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Because of the orthotropy of the tile material, lack of material data for a better model for probability of failure in the orthotropic tile materials, and time constraints, Eq. (6) was reduced to integration over each element using only the orthotropic stresses at the Gaussian quadrature points.

4. Numerical Modeling

The primary tool for mathematical modeling was a 2-dimensional Finite Element computer program, TPS-1, with *probability of failure* calculation incorporating Weibull statistics. The program is linear, 2-dimensional, orthotropic material, Finite Element program with default values of mesh and material parameters designed for a standard acreage tile. The program is interactive for easy change of dimensions and loads and material parameters for both tile and SIP; has options for easy plotting of stresses and/or the P.O.F. calculation, and is optimized for computer efficiency on the CDC computer. The program uses isoparametric four-node quadrilaterals. The material properties can only vary within groups of elements. with 4 possibly different tile materials, 10 SIP materials and 8 filler-bar materials. In order to simulate the non-homogeneous material through the tiles, orthotropic material properties and orthotropic Weibull parameters are defined randomly for each element by a uniform distribution function between specified upper and lower limits for each quantity. A representative finite element subdivision of the tile-SIP system is shown in Fig. 5.

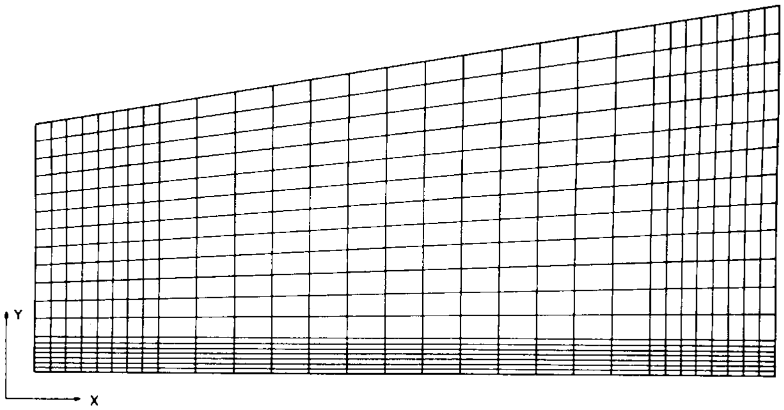

Fig. 5. Tile Slice Mesh Configuration

Based on the arguments of Section 3 above, the probability of failure is computed according to the relation:

$$\text{P.O.F.} = 1 - \exp(-B) \qquad (7)$$

where B is computed as

$$B = \sum_{k=1}^{N_e} \iiint_{V_k} \left(\frac{1}{\lambda_k}\,!\right)^{\lambda_k} \left[\left(\frac{\sigma_x}{\sigma_{ox}}\right)^{\lambda_k} + \left(\frac{\sigma_y}{\sigma_{oy}}\right)^{\lambda_k} + \left(\frac{\sigma_z}{\sigma_{ox}}\right)^{\lambda_k}\right] dV \tag{8}$$

The summation in Eq. (8) is over the elements of the finite element model and the volume integral is computed numerically for each element. The material parameters σ_{ox} and σ_{oy} are the in-plane mean failure stresses and λ_k is an average Weibull Modulus for the orthotropic directions. Because of the 2-D limitations of the program a "strip" approximation was used. The substrate deflections have been evaluated separately for each strip position, and each strip was loaded according to its position and the given loads. Then the contribution to the B-integral in the P.O.F. calculation was determined using TPS-1. These were then summed for the strips, considering the volume of the strip and the final P.O.F. values were determined. A stepwise loading with the secant modulus of elasticity for each step was used to approximate the nonlinear SIP behavior.

5. Results and Discussion

Probability of failure results, P.O.F., were calculated for the SIP/Tile system under a uniform load on the tile, for both densified and undensified tile, using the material properties and orthotropic Weibull parameters given in Table 1. The σ_x^o value in the boundary layer is reduced to account for stress concentration from SIP fiber bundles analysis. The results are given in Figure 6. These results show a significant increase in allowable loading for a given P.O.F. associated with densification or, conversely, a significant decrease in P.O.F. for a given uniform nominal loading for the densified system. Results are also shown for Weibull Moduli of $\lambda=6$ and $\lambda=10$ for both the undensified and densified tile systems. The P.O.F. values are seen to be sensitive to the Weibull parameters and since the Weibull parameters are not well established for either undensified or densified, the actual value of P.O.F. may not be as meaningful as their relative values in comparing different effects such as the effect of substrate mismatch, filler bars, instrumentation cutouts, etc.

Three tiles from a Body Flap Test panel for which loading information was made available are shown in Figure 7. Weight information, tile "g"

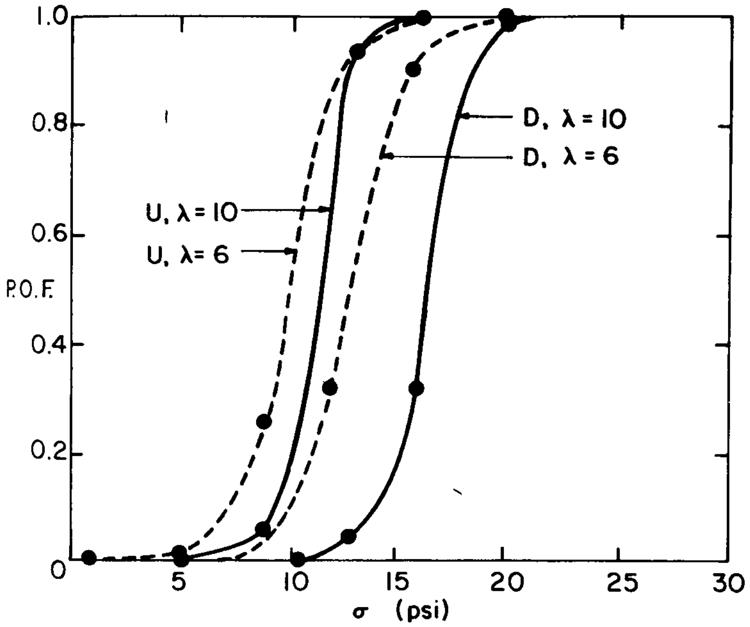

Fig. 6. Results with Volume Correction for Full SIP/Tile Unit.

loading and coefficients for eccentric moments were furnished by NASA and are summarized in Table 2.

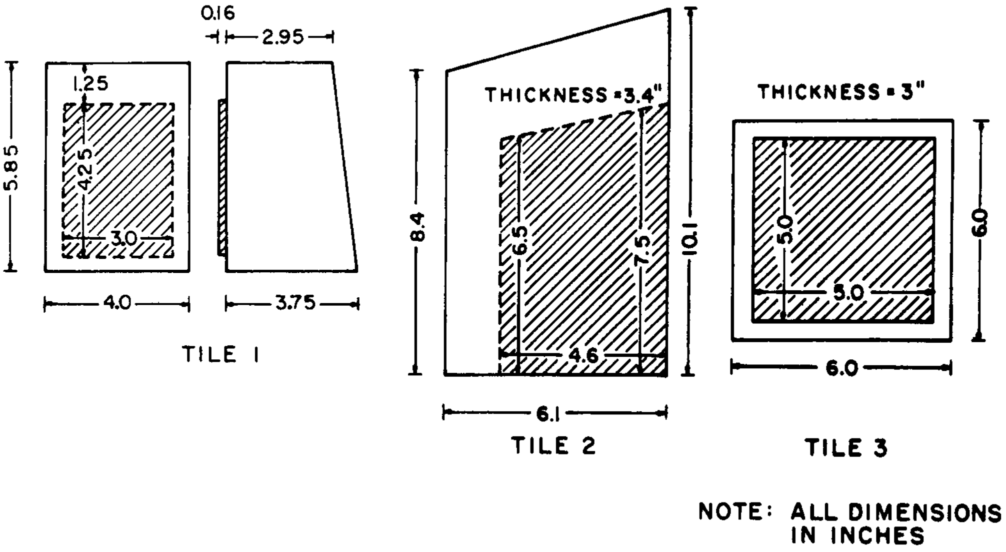

Fig. 7. Body Flap Test Panel Tiles.

Table 2

Body Flap Test Panel Tile Loadings (See Figure 7)

Tile No.	Weight (lbs.)	"g" loading	Vert. load w x g (lbs.)	Moment coefficient* about horiz. axis	Moment coefficient* about vert. axis
1	.52	95	49.4	.875	0
2	1.25	95	118.8	1.04	.715
3	.63	144	90.7	0	0

*Moment about cg of SIP pad = coeff. x vert. load.

The results of the analysis using TPS-1 are summarized in Table 3 for Weibull Moduli of λ=6 and λ=10 using for undensified LI-900 σ^o_y = 12 psi, σ^o_x = 120 psi. Results are also given for λ = 6, σ^o_y = 10 psi as a probable reduction for vibration loading. Results are also given for Tile No. 3 for a loading corresponding to a nominal SIP stress of 9.3 psi for comparison with results available from proof testing by Rockwell International.

Table 3

Summary - Application to Body Flap Test Panel Results

	Probability of Failure (σ^o_y = 12 psi, σ^o_x = 120 psi)			*Values using $\sigma^o_{y_{dyn}}$ = 10 psi
Weibull Moduli	Tile #1 (eccentric) @ 95 g	Tile #2 (corner) @ 95 g	Tile #3 (acreage) @ 144 g	Tile #3 (acreage) @ 9.3 psi nominal SIP
λ=6	0.13 [.34]* (13%)	0.43 [.81]* (43%)	5.5×10^{-3} [.016]* (0.6%)	0.703 (70%)
λ=10	0.062 (6%)	0.12 (12%)	0.04×10^{-3} (.004%)	0.27 (27%)
Comment	Failed	Failed	No Failure	Proof Test Failure Rate 0.714 (71.4%) at 9 psi
	Dynamic Tests			Static

Since the Weibull parameters were not well established, the Weibull values determined at the University of Washington were combined with data from proof testing of actual tile by Rockwell International to obtain the values in Table 3. The Rockwell data indicated a σ_y^o value of about 12 psi for static testing and a Weibull Moduli, λ, of about 6 (see last column of Table 3), the σ_x^o value of 120 psi comes from UW data. These values applied to the three Body Flap Test panel tile give the P.O.F. values of 13%, 43% and 0.06% in Table 3. Since the Body Flap Test was a vibration test with repeated loading, it is expected that the Weibull values should be further reduced based on the number of cycles of loading at the high stress level. For example, if σ_y^o is reduced to 10 psi, the P.O.F. results (with an asterisk) in Table 3 of 34%, 81% and 1.6%, respectively, are obtained for the three tiles. The high P.O.F. values for tile 1 and 2 are consistent with the observed failure and the low value for tile 3, with no failure. Additional statistical data and test results of this kind should allow refinement of the analysis methods and Weibull parameters so that this is a reliable approach to prediction of performance of the tile.

Some additional data for the determination of Weibull parameters was made available in March 1981 from tests by the NASA-Langley structures team on W-3 tiles and MF-5 densified tiles. The tiles were pulled to failure to determine the residual strength after being subjected to static "proof test" loads and mission fatigue test loadings. Weibull parameters were determined from the residual strength and are presented in Table 4.

Table 4

Weibull Parameters from NASA-Langley - Residual Strength of W-3/MF-5 Tiles (after simulated 72 ascent mission fatigue tests)

Region	No. of Tiles	Max. Static Loads kPa (psi)	σ_0 kPa (psi)	λ
W-3	8	44.1 & 50.33 (6.4 & 7.3)	87.6 (12.7)	8.1
MF-5	9	20.7, 33.1 & 37.5 (3.0, 4.8 & 5.5)	100.7	10.7
Combined	17	20.7 to 50.3 (3.0 to 7.3)	55.2 (13.8)	8.4

The data on which Table 4 is based are the only values made available to the authors incorporating any contribution of mission loading to the Weibull data base. Unfortunately, comparable strength values for tiles which had not been subjected to fatigue testing were not available to provide virgin Weibull parameters.

Using this Weibull data base the above analysis is applied to the tile on the "Columbia" for expected load and moment vectors for its maiden flight on April 12, 1981.

Only the results for a standard acreage tile with the largest force and moment vectors and substrate deflection of those furnished by NASA/ Langley are presented here. These were for tile O3-191009-144 and the quantities were as follows:

$$\overline{F} = -7.12\ \overline{x} + 1.13\ \overline{y} - 19.19\ \overline{z} \qquad \text{kg}$$

$$(\overline{F} = 15.7\ \overline{x} + 2.5\ \overline{y} - 42.3\ \overline{z}) \qquad \text{(lbs)}$$

$$\overline{M} = -0.19\ \overline{x} - 4.11\ \overline{y} \qquad \text{m-N}$$

$$(\overline{M} = -1.7\ \overline{x} - 36.4\ \overline{y}) \qquad \text{(in-lbs)}$$

acting at the c.g. 20.82 mm (0.82 in) from the base of the tile. The substrate deflections were:

0.30 mm (0.0117 in) amplitude warpage with a 1/2 wavelength 246.38 mm (9.7 in).

0.10 mm (0.005 in) amplitude out-of-plane deflection with a 1/2 wavelength of 52.07 mm (2.05 in).

The tile was assumed to be 152.40 mm x 152.40 mm (6 in x 6 in) with a 1.27 mm x 1.27 mm (5 in x 5 in) SIP footprint. The same loading and substrate deflections were used for DFI (instrumentation) tile of the same size with an assumed 44.45 mm (1.75 in) diameter SIP cut-out at the center of the SIP. Results were obtained for densified and undensified tile for both the acreage tile and the DFI tile. The Weibull parameters were chosen based on the previous values and on the "conditioned" values of Table 4 from residual strengths after simulated mission fatigue tests. The Weibull values used for the P.O.F. values reported here are in Table 5. Using these Weibull parameters the P.O.F. values of Table 6 would be obtained if densified and undensified LI-900 tiles with and without an instrumentation cut-out were in positions to be subjected to the above defined loading and substrate deflection expected to occur on the STS-1 mission. The values presented are the highest values from the separate strip theory approxima-

Table 5

Weibull Parameters for LI-900 - STS-1 Mission Loading

	Undensified Tile		Densified Tile	
	Virgin	"Conditioned"	Virgin	"Conditioned"
kPa σ_o (psi)	93.1 (13.5)	93.1 (13.5)	165.5 (24)	165.5 (24)
λ	10	8.5	10	8.5

Table 6

P.O.F. Values from Calculations for LI-900 STS-1 Loading

	Acreage Tile		DFI Tile	
	Undensified	Densified	Undensified	Densified
Virgin	0.5×10^{-5}	0.3×10^{-5}	0.4×10^{-4}	0.25×10^{-4}
"Conditioned"	0.6×10^{-4}	0.3×10^{-4}	0.8×10^{-2}	0.5×10^{-3}

tions. The results in the table indicate that in the range of loadings and P.O.F. values considered here, the P.O.F. is almost halved in all cases by densifying, but they are increased more than an order of magnitude by the "conditioning" associated with the above mission fatigue test data. In the worst case of undensified DFI tile the P.O.F. is increased 200 times for the "conditioned" compared to the virgin SIP. The P.O.F.'s are generally increased about an order of magnitude or more by the instrumentation cutouts.

The results obtained here are believed to be qualitatively significant and correct in indicating the effects of modifications and of deterioration associated with mission loadings, even if the present material property data base and load data are such that both the loadings and Weibull parameters must be corrected for the actual STS-1 tiles and mission loadings.

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