mputers & Structures Vol. 18, No. 4, pp. 575-581, 1984
Printed in Great Britain.

0045-7949/84 $3.00 + .00

MODELLING BOAT WAKE LOADING ON LONG FLOATING STRUCTURES

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(*Received* 21 *March* 1983; *received for publication* 10 *May* 1983)

Abstract—A model for the boat wake loading on long floating structures is presented. The loading is a train of short duration waves moving along the structure. Basic characteristics of the loading profile are obtained from existing literature after certain simplified assumptions. The loading model is implemented numerically obtaining the nodal force time series in a finite element analysis. Computer subroutines and example of applications are shown.

1. INTRODUCTION

Floating breakwaters built to protect marinas are usually quite flexible, because of the small wind generated waves at the restricted fetch regions of the marinas. The new type of breakwater introduced by the U.S.A. Army Corp of Engineers, made up of rigid pontoons about 75 ft long connected with flexible rubber connectors, is a typical example of a flexible breakwater. In many cases it is usual that the structural response due to boat wake, from vessels passing close to the breakwater, is larger and more critical in the design than the response to wind generated waves. Some breakwaters are damaged from the waves generated by vessels passing near, rather than from wind generated waves, since in the lapsed time of the breakwater extreme values have not been reached.

A design approach of long floating structures to boat wakes is therefore desirable. The usual method of design is to form a finite element model of the structure and perform dynamic analysis under the wave excitation. The only part of the analysis, which needs investigation is the evaluation of suitable nodal loads. The first step is to evaluate the character of the loading and second step is an analytical transformation of the loading to nodal force time series and the numerical implementation to a finite element program.

The quantitative evaluation of the loading considers the character of the boat generated waves as a function of ship type and size, speed and water depth. A considerable amount of published work exists on ship generated waves, but most of them consider mainly the wave resistance of the ship and a few are concerned with the basic quantitative characteristics of the generated waves.

This paper deals with the development of a model for boat wake loading on the structure. Based on the existing literature a simple model for the wave forces on the structure will be developed and it will be implemented in the nodal forces. The numerical implementation in finite element programs for the dynamic analysis of the structure and appropriate computer subroutines will be shown. Examples of practical applications of the response of floating breakwaters will be presented.

2. BOAT GENERATED WAVES

A theory explaining the generated wave pattern by a moving point in deep waters has been developed by Lord Kelvin[1, 2]. This theory has been developed further, and supplemented with experimental measurements for the generated waves by a moving vessel in deep and shallow waters[3-10]. Here we present in summary the most important and necessary aspects of the theory describing the wave pattern.

When a ship moves with a speed V it generates two sets of waves with its bow and stern, diverging and transverse waves (Fig. 1). The crests of these two sets of waves meet at a common tangent along a line approx. 19° from the sailing direction. This line is a focus of the cusp points where the wave height is maximum. The wave direction at the cusp is approx. 55° from the sailing direction. The wave height depends on the ship type and size, speed and water depth, and it decreases significantly with the distance from the ship. References [8-10] are a good source of information for an estimate of the wave height for certain types of boats. The wave lengths for the two wave systems are

$$\text{Transverse waves: } \lambda_t = \frac{2\pi(V)^2}{g}\sec^2\theta \qquad (1)$$

$$\text{Diverging waves: } \lambda_d = \frac{2\pi(V)^2}{g}\cos^2\theta \qquad (2)$$

where θ is the wave angle from the sailing line. The diverging waves are generally higher than the transverse ones and of shorter wave length. At greater distances from the sailing line the diverging waves are more dominant than the transverse waves. In this work the modelling of the wave forces on the structure will be based on the diverging waves.

3. BOAT WAKE LOADING

Here we will address the problem of obtaining a reasonable and easily implemented numerical model of boat wake loading on long floating structure. For this we will use the arguments of the previous paragraph and certain simplified assumptions to reduce the complexity of the problem. Figure 2 shows the general definition and nomenclature of the problem.

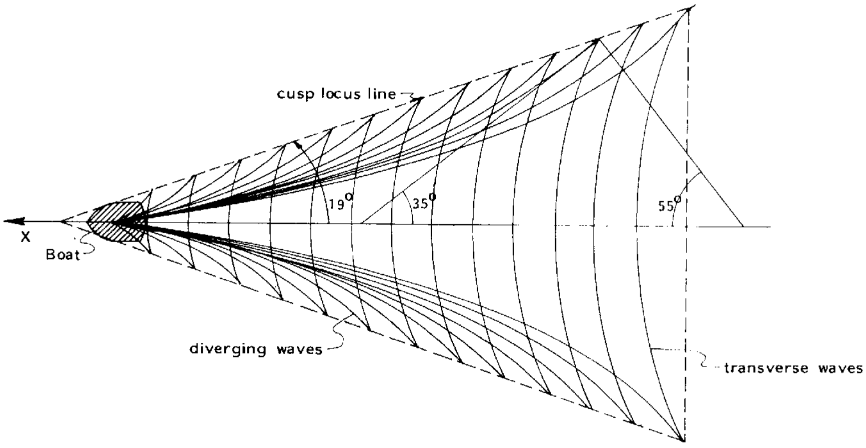

Fig. 1. Boat generated waves.

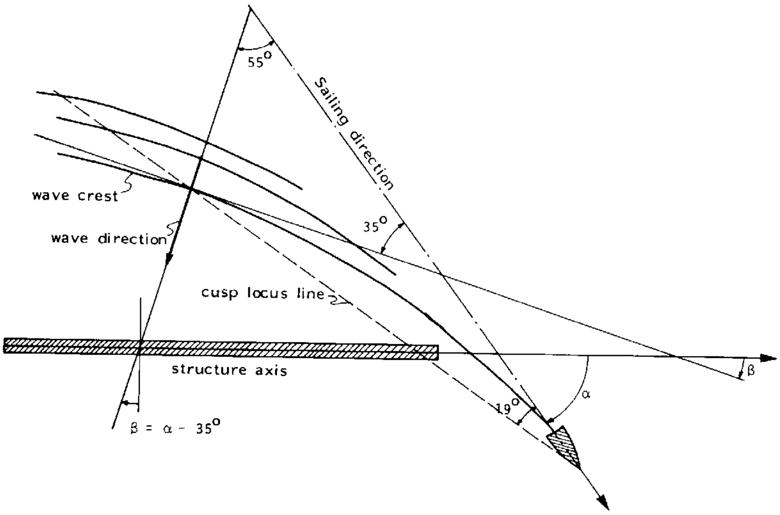

Fig. 2. General definition and nomenclature.

Given a boat sailing at an angle α with the axis of the structure, and with a speed V, the question then is to define the resulting loading on the structure.

A basic assumption which we are going to follow here is that the wave action is concentrated at the cusps which lie on a line at an angle approx. 19° from the sailing line. The wave direction at this point is approx. 55° with the sailing line or at an angle $\beta = (\alpha - 35°)$ with the structure axis. The result is a wave loading on the structure, as the one shown in Fig. 3, at the point of intersection of the cusp focus line with the structure axis.

From simple geometric relations (Figs. 2 and 3), we obtain that this wave loading is moving along the structure axis with a velocity v related to the boat velocity V as

$$v = -V\frac{\sin 19°}{\sin(\alpha - 19°)}. \quad (3)$$

For this relation the positive directions for the sailing and the structure axis are as shown in Fig. 2. Figure 4 shows a graph of eqn (3) for various boat directions of sailing.

The additional characteristics of the loading are obtained as has been discussed in the previous paragraph. The wave period will be obtained from the wave length using eqn (2), and $\theta = 55°$.

$$T = \frac{2\pi}{g} V \cos 55° = 3.60\frac{V}{g} \quad (4)$$

(g: acceleration of gravity).

4. MODEL LOADING, NODAL LOADS

The final model of the boat wake load consists of a train of waves (Fig. 5), moving along the structure axis with velocity v obtained from eqn (3). The profile of each of the individual waves should be of one main

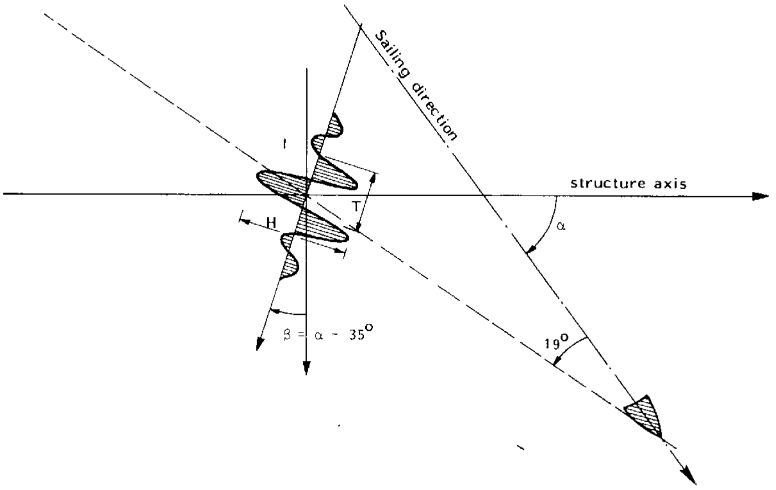

Fig. 3. Boat wake loading.

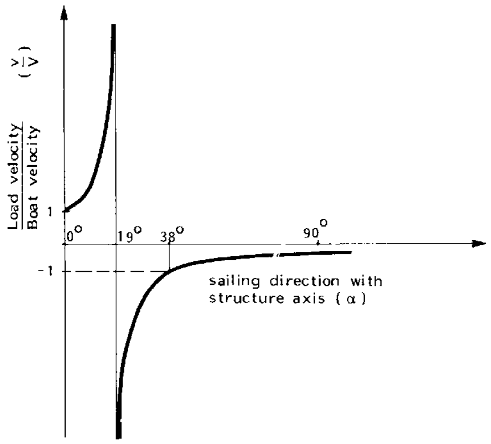

Fig. 4. Loading velocity along the structure.

wave of height H preceeded by some waves of increasing amplitude and followed by some waves of decaying amplitude. A shape shown in Fig. 6 and described by the following relation

$$\eta(t)=\frac{H}{2}\sin\left(2\pi\frac{t}{T_m}\right)\sin\left(2\pi\frac{t}{T}\right) \tag{5}$$

has been chosen as a first approach to the problem. There is very little information in the literature on the number of waves for the profile of this model. From some observed preliminary measured data a ratio T_m/T of 6–18 (Fig. 6), seems appropriate.

Assuming that at time $t = 0$ the waves just struck the left end of the structure then at time t the wave disturbance is as shown in Fig. 5. The wave forces at a point on the structure are

$$f(x,t) = \delta(\omega,\beta)\left(\frac{H}{2}\right)\sin\left(\frac{2\pi}{T_m}\frac{x_A-x}{v}\right)\sin\left(\frac{2\pi}{T}\frac{x_A-x}{v}\right) \tag{6a}$$

$$\text{for } x_B \le x \le x_A$$

$$f(x,t)=0 \quad \text{for } x<x_B \text{ or } x>x_A \tag{6b}$$

where

$$x_A = vt \text{ and } x_B = v\left(t-\frac{T_m}{2}\right). \tag{6c}$$

The coefficient $\delta(\omega,\beta)$ is the hydrodynamic coefficient converting the wave amplitude to wave force on the structure. It is wave frequency (ω) and wave direction (β) dependent. Tables and graphs for these hydrodynamic coefficients can be found in Refs. [11, 12].

For a finite element model of the structure (Fig. 7), it is necessary to obtain the nodal loads. Assuming a displacement formulation with linear displacement fields we have for the displacement field of node i

$$N_i(x)=\frac{x-x_{i-1}}{x_i-x_{i-1}} \quad x_{i-1}\le x\le x_i \tag{7a}$$

$$N_i(x)=\frac{x_{i+1}-x}{x_{i+1}-x_i} \quad x_i<x<x_{i+1} \tag{7b}$$

$$N(x)=0 \quad x<x_{i-1},\ x>x_{i+1}. \tag{7c}$$

Applying virtual work the nodal loads are obtained as

$$R_i(t)=\int_0^L N_i(x)f(x,t)\,\mathrm{d}x \quad (L = \text{bridge length}) \tag{8}$$

where $R_i(t)$ is the load time series at node i. Equation (8) can be integrated explicity for the nodal loads.

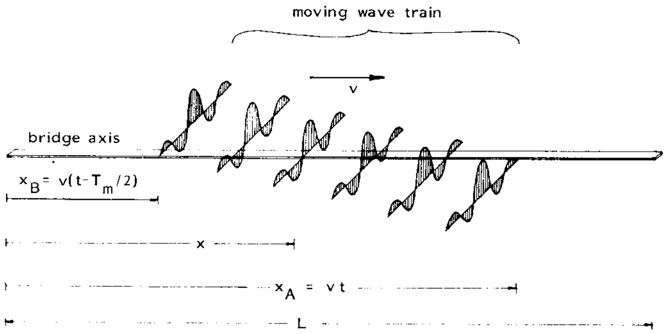

Fig. 5. Boat wake loading along the structure.

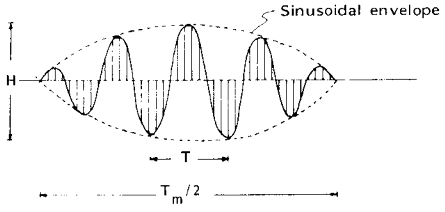

Fig. 6. Model of loading profile ($T_m/T = 9$).

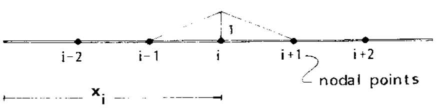

Fig. 7. Finite element model, linear displacement field.

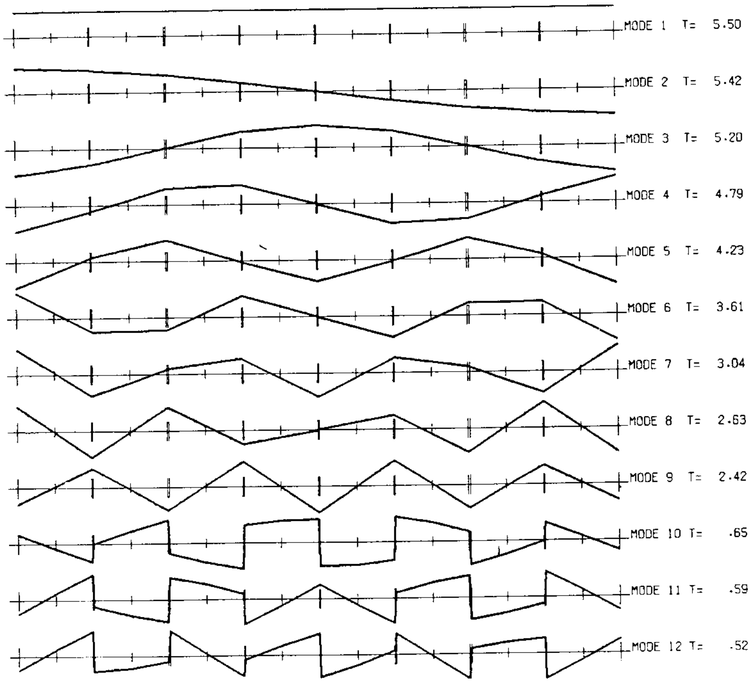

Fig. 8. Example structure. Mode shapes and natural periods.

5. NUMERICAL IMPLEMENTATION APPLICATIONS

Using eqn (8) together with eqns (6) and (7) the nodal loads can be obtained for each time point, from $t = 0$ when the disturbance is at one end of the structure until $t = L/v$ when the disturbance is at the other end of the structure. After the time series for the nodal loads have been constructed a time domain dynamic analysis can produce the response values. A set of computer subroutines to do the job is presented in Appendix A together with information showing the way they can be included in a finite element program.

The method described above has been implemented in a computer program for the analysis of floating bridges and breakwaters[11, 13]. As an example a boat wake load will be applied at a typical breakwater with flexible connectors, like the ones of the U.S.A. Army Corp of Engineers, and results obtained with the above mentioned computer program will be presented. The breakwater is 800 ft long, and made up of eight identical pontoons connected with flexible rubber connectors. Information about cross section properties, anchor cables, connector properties, etc. can be found in Ref. [11, 13]. The natural frequencies and structural mode shapes are shown in Fig. 8. The boat sails at an angle $\alpha = 44°$ with the breakwater, and with a speed 19.0 m.p.h. From eqn (4) we obtain for the wave period of the loading $T = 3$ sec. The wave height has been taken $H = 1.0$ ft. The load travels with a speed along the breakwater axis obtained from eqn (3), $v = 14.6$ m.p.h. A ratio $T_m/T = 6$ has been chosen. Figure 9 shows maximum values for the displacements, bending moments and shearing forces along the breakwater. Figure 10 shows representative time series for the response values.

6. CONCLUSIONS

A model for boat wake loading on long floating structures has been presented. The model can be

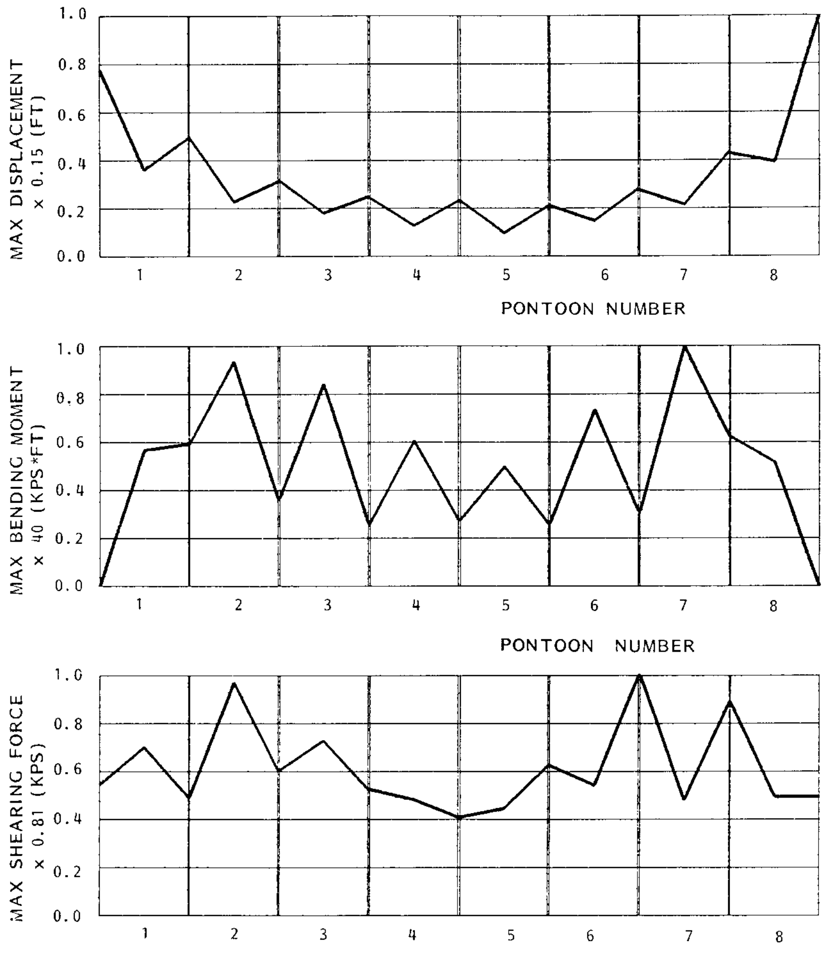

Fig. 9. Example structure. Maximum response values.

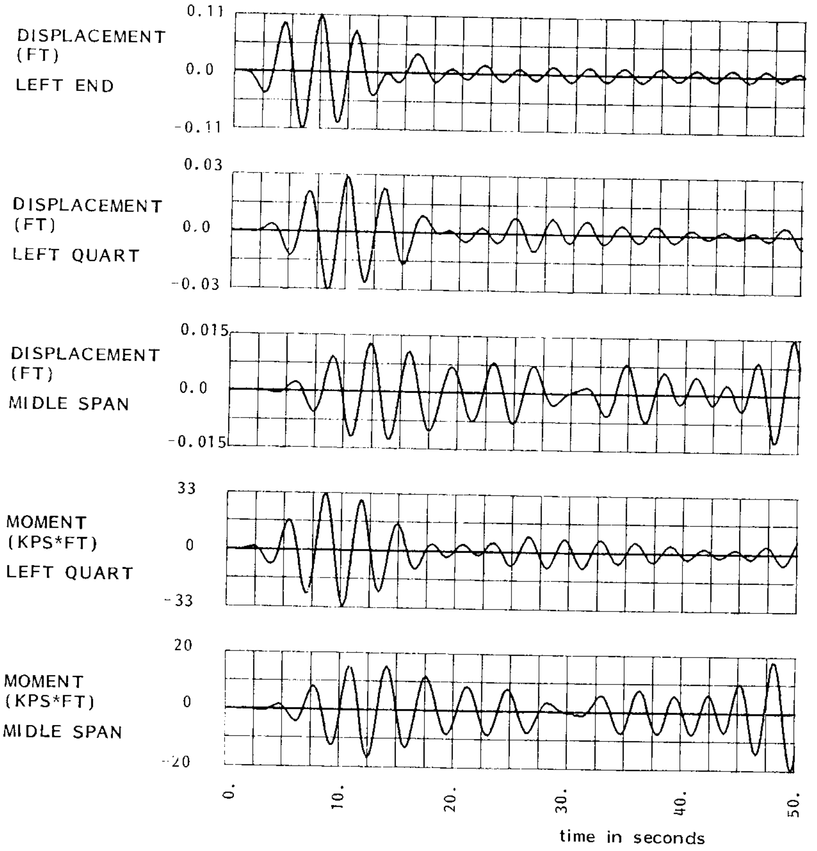

Fig. 10. Example structure. Representative time series of response values.

easily implemented in a finite element program for the dynamic analysis of the structure. Some simplifications have been used to overcome the lack of informations and experimental data in obtaining the model loading. More in field and experimental measurements will help to calibrate the various parameters of the model as well as to verify its accuracy and improve it.

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