CGFLOAT (A computer program for the dynamic analysis of floating bridges and breakwaters)

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This paper describes the program CGFLOAT, a finite element program to calculate the dynamic response of floating bridges and breakwaters in short-crested sea states. The response can be calculated via a time or a frequency domain analysis. Special methods have been incorporated for simulating the nodal loads from a directional wave spectrum. The program is applicable to any kind of pontoon arrangement, stiff or flexible connections between the pontoons, variations of anchoring systems, hydrodynamic forces, and sea state conditions. The effort for preparing the input data has been minimised, and the format of the output data has been optimised to include all the necessary information for design or analysis concerning the stochastic characteristics of the loading. The program is a self-contained package with small memory requirements, it has the option of digital or continuous graphical output, and it is machine independent.

Key words: floating bridges, floating breakwaters, wave loading, short-crested waves.

INTRODUCTION

Floating bridges and breakwaters are special kinds of structures for design and analysis. Structural modelling using finite element methods does not present any difficulties, but the part of the analysis which presents special problems is the modelling of the loading in a short-crested sea.

For the response calculations general purpose finite element programs can be used. These programs do not provide methods for calculating the loading in a stochastic short-crested sea, and additional routines should be included in order to do the job. General purpose finite element programs are not tailored for a particular problem, their use is time consuming, uneconomical, and susceptible to errors. This is because special pre- and post-processors for the data and additional routines to calculate the loading need to be written, and many transformations and manipulations of data series will take place. In addition, for the response calculation the usual frequency domain analysis is quite expensive as the cross-spectra matrices are not diagonal in the case of a short crested sea loading.

This program has been developed especially for long floating structures in short crested sea loading. The computer code has been optimized taking into account the special characteristics of the structure and the loading. Theories and methods for hydrodynamic loading, short-crested waves, wave spectra, sea state simulation, and stochastic dynamics are included in the calculation routines. The amount of input data has been reduced to a minimum, and the format of the output data has been optimised so most of the information is obtained in a graphical convenient form.

For the response calculation a Monte-Carlo simulation is used. This method is considered to be more advantageous over the usual frequency domain analysis, which is the only alternative, because it reduces the computational cost considerably and can be used for frequency and time domain analysis. It is based in simulating sets of nodal load series and calculates the structural response by deterministic dynamic analysis. The expected response values are obtained in the end by calculating the ensemble statistics between the simulated cases. The basis for computing the sets of nodal load series is the wave coherence along the structure, which is obtained from the directional wave spectrum.

PROBLEM DEFINITION THEORETICAL TREATMENT

The objective of the program is to compute the dynamic response of long floating structures in a short crested sea.

The equation of motion of the structure is

\[ m \ddot{f}(t) + c \dot{f}(t) + k f(t) = R(t) \]  

(1)

where the mass and damping matrices, \( m \), \( c \), include the hydrodynamic part, \( k \) and the stiffness matrix, \( k \), includes the buoyancy effects.

The three directions of motion have been considered uncoupled. The small coupling between sway and roll motion due to the hydrodynamic forces has been neglected. This reduces the computational cost considerably without affecting significantly the accuracy of the results. Finite beam elements of half pontoon length, linear elastic springs for the lateral anchoring and consistent mass matrices are used in the structural modelling. Flexible or rigid connectors between the pontoons can be specified. The hydrodynamic coefficients are frequency dependent in the frequency domain analysis and constant in the time domain analysis.

The load vector \( R(t) \) is obtained from the wave field characteristics, which is usually described by a directional spectrum giving the wave energy distribution over frequencies and directions. The nodal loads are simulated via a Monte-Carlo simulation from the directional spectrum. To take care of the stochastic characteristics of the loading, a
number of nodal load sets are simulated, the structure response is calculated for each set of nodal loads, and the expected response values are obtained from the ensemble statistics.

The basis for the simulation of the nodal loads is the wave coherence between two points along the structure. The wave coherence can be obtained directly from the directional spectral model, and it is a decaying function with the wave length. An exponentially decayed coherence of the form

$$\gamma_w \left( \frac{\Delta \xi}{\lambda} \right) = \exp \left( - \alpha \left( \frac{\Delta \xi}{\lambda} \right)^2 \right)$$  (2)

is a satisfactory approximation for most practical applications. The coefficients \(\alpha\) and \(\beta\) can be found in tables in refs. 1 and 3 for various models of directional coherence.

Implementing the wave coherence in the nodal loads we obtain

$$|S_{R_j R_j}(\omega)| = \delta^2(\omega) \rho_{\phi}(\omega) S_{W}(\omega)$$  (3)

where \(|S_{R_j R_j}(\omega)|\) is the norm of the cross-spectral nodal load matrix; \(\delta^2(\omega)\) is a frequency dependent hydrodynamic coefficient converting the wave amplitude to force waves along the structure and can be found in tables in ref. 2 for various directional spectral models, \(S_{W}(\omega)\) is the unidirectional wave spectrum; and \(\rho_{\phi}(\omega)\) depends on the wave coherence and nodal point spacing

$$\rho_{\phi}(\omega) = \sum_{i=1}^{2} \sum_{j=1}^{2} L_{ij} L_{ij} \int_{0}^{1} \int_{0}^{1} N_{i}(\xi) N_{j}(\eta) \xi d\xi d\eta$$  (4)

For equation (4) the notation of Fig. 1 is used, \(i_1\) and \(i_2\) are the spans left and right of node \(j\), and \(j \in J\) are the spans left and right of node \(j\). \(\xi\) and \(\eta\) are normal coordinates corresponding to spans adjacent to nodes \(i\) and \(j\), \(\Delta \xi_{i j} \) is the distance from a point in span \(i_2\) to a point in span \(j_1\), \(N_{i}(\xi)\) and \(N_{j}(\eta)\) are displacement functions for spans \(i\) and \(j\), \(\lambda\) is the wave length. \(\lambda = 2 \pi / \omega^2\). The \(\rho_{\phi}(\omega)\) functions are calculated using a Gauss four point integration scheme, third degree polynomials for displacement functions, and \(\gamma_w\) function of the form of equation (2).

The nodal loads \(R_{j}(t)\) are simulated with the help of a set of \(N\) uncorrelated load series, \(X_{j}(t), X_{j}(t), . . . , X_{j}(t)\), having

$$S_{X_{i} X_{j}}(\omega) = \begin{cases} \delta^2(\omega) S_{W}(\omega) & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$  (5)

as

$$\mathbf{R}(t) = \mathbf{a} \mathbf{X}(t)$$  (6)

From equations (6), (5) and (3) and for narrow band

$$\mathbf{R}(t) = \mathbf{a} \mathbf{a}^T = \mathbf{p}$$  (7)

where \(\mathbf{a}\) is the matrix of \(a_{ij}\), and \(\mathbf{p}\) is the matrix of \(p_{ij}\) evaluated for \(\omega\) equal to the mean wave frequency.\(^4\) Equation (7) can be solved for \(\mathbf{a}\) using the eigenvalue matrix \(\mathbf{A}\) and the eigenvectors \(\Phi\) in respect to a unit matrix as

$$\mathbf{a} = \Phi \Lambda^{1/2} \Phi^T$$  (8)

After the simulation of the nodal forces the solution of the dynamic equations of motion proceeds in the usual deterministic way for the time domain analysis. Mode shape superposition is used in the program for more economic solution as many high mode shapes do not participate in the response.

For the frequency domain analysis the frequency components of the force series \(X_{j}(t)\) are considered

$$\mathbf{X}(t) = \sum_{k=1}^{M} \mathbf{c}_k \cos (\omega_k t + \phi_k^j)$$  (9)

where \(c_k^j\) are amplitudes calculated from the load spectral density; and \(\phi_k^j\) are random phase angles between 0 and \(2\pi\). The nodal forces are obtained using equation (6) as

$$\mathbf{R}(t) = \sum_{k=1}^{M} \sum_{i=1}^{N} a_{ij} \cos (\omega_k t + \phi_k^i)$$  (10)

Simplifying equation (10) one step further, for the response calculation in the frequency domain, the structure is loaded with \(M\) sets of harmonic components \((k = 1, 2, \ldots, M)\) at the nodes

$$\mathbf{R}(t) = \sum_{k=1}^{N} \mathbf{a} \mathbf{c}_k \cos (\omega_k t + \phi_k)$$  (11)

where \(\phi_k^j\) are random phases between 0 and \(2\pi\). The final response is computed by superimposing the \(M\) harmonic components.

To obtain expected values for the response of the structure a number of sets of nodal loads is simulated using different sets of \(X_{j}(t)\) series. After the response calculation to the various nodal load sets, ensemble statistics between the results produce the expected response values. The number of the simulations is important in the accuracy of the final results. The optimum number of simulations can be obtained looking at the improvement in the accuracy of the resulting expected response values as the number of simulations increases. A number of simulations between 8 and 16 has been proven from experience to be adequate for stable responses.\(^1\)

**PROGRAM CAPABILITIES**

The problem, which the program solves, is the interaction between structure and water environment. The program combines fluid, structural, and stochastic process theories in one program. Thus the response computation of long floating structure is reduced to a routine analysis.

The following aspects have been implemented in the program:

1. Response analysis of straight floating bridges and breakwaters, for the three directions of motion, sway, heave, and roll.
2. Modelling of continuous structures and structures with flexible connectors between the pontoons.
3. Eigenvalue solution, time and frequency domain analysis.
5. Boat wake analysis.
6. Monte-Carlo simulation of the wave loading.
7. Statistical evaluation of the results for the simulated response.
8. Frequency dependent hydrodynamic coefficients.
9. Different kinds of units can be specified.
10. Graphical output, and results in a convenient format.

INPUT

To minimise the effort of the analyst or designer great care has been taken to reduce the amount of input data. Fig. 2 shows the input data for a typical case of a breakwater with flexible connectors. Following the numbering on the side of the figure, the basic sets of supplied data are shown.

The geometry of the structure and the nodal points are automatically generated by the program after the basic pontoon properties are supplied. The hydrodynamic coefficients are input for certain wave frequencies and the program interpolates between them for the necessary values in the response calculations.

The short-crested sea state is specified by the wave spectrum and the wave coherence. Typical wave spectra like Pierson-Moskowitz, JONSWAP, Darbichire-Scott, are computed from their parameters by the program. Other kinds of spectra can be read in directly. Wave time series are simulated from the spectra, via various methods, by the program, or can be read in from existing disc files. The wave correlation is handled by specifying an exponential wave coherence, equation (2), or by specifying the nodal load correlation directly. In the case of boat wake analysis the speed and characteristics of the boat wake are specified.

For the frequency domain analysis, the frequencies for the computation of the frequency response function can be specified or they are computed by the program after their number is specified. For the time domain analysis, participating modes, integration method and parameters, accuracy, time interval and time length are specified.

The user specifies the various analysis paths, units, simulation methods and number, and required printed or plotted output quantities. For all the cases if general parameters are not specified by the user, default values assigned by the program. The default values are either values more often used or values which optimise the solution. A typical set of input data can be prepared in about 10 to 15 min.

OUTPUT

The results from the frequency or time domain analysis is a large number of values. To be useful they are outputted in graphs. The graphs are dependent on the plotting capabilities and the computer software. For this reason two versions of the program have been developed. In the first version the plots are in digital form and additional library subroutines are not needed except the usual FORTRAN IV. In the second version the plots are in continuous form and the CALCOMP routines are needed together with graphical output devices. The output from both versions is adjusted to a standard page format.

A typical output of the program contains:

1. A printout with all the information of the structure, the wave field, assumptions and parameters used.
2. Printout of the eigenvalues.
3. A plotted output with the structure properties, wave spectra, wave series, etc. (Fig. 3).
4. Plots of mode shapes (Figs. 4 and 5).
5. Plots of the structure response to unit amplitude short crested waves of various frequencies.
6. Plots of ensemble maximum, mean, and standard deviation values, between the simulated responses, for displacements, bending moments, and shearing force, along the structure (Figs. 6 and 7).
7. Sample plots of time series of the response at various places of the structure (Fig. 8).

SOFTWARE

The program is written in FORTRAN IV. It is a dynamic code allocating storage in the blank COMMON as it is required during the execution. It uses the minimum necessary memory size. The special character of the structure has been taken into account when the matrices are formed. The rotational degrees of freedom are condensed and the stiffness and mass matrices are stored and used in banded form.

The reading and writing operations from low-speed storage have been reduced to the minimum possible. The maximum possible number of data is kept in-core. In case the data do not fit the high-speed storage, as in the case of time domain analysis, they are stored in low-speed disc storage in blocks. The number and size of the blocks are calculated by the program.

A memory size of 4n² words, where n is the number of nodal points, is enough for the program blank COMMON needs. For commonly encountered structures the number of nodal points used to obtain good results is about 30 to 40, which corresponds to a blank COMMON of 3600 to 6400 words long. In case there is more memory available in the computer the program will use it effectively and the execution time will be reduced.

The execution time depends on the number of the calculated results, like eigenvalue solution, frequency domain analysis, time domain analysis, the number of nodal points, the number of simulated responses in order to compute ensemble statistics, the number of frequencies in the frequency domain analysis, the number of time points and the participating modes in the time domain analysis, and the size of the available memory. Typical execution

Figure 3. Structural properties and sea state data

Figure 4. Mode shapes, continuous plots

Figure 5. Mode shapes, digital plots
Figure 6. Response values, continuous plots

Figure 7. Response values, digital plots

Figure 8. Sample time series plots

time in a CDC 6600 for an average structure ten pontoons long and for a reasonable number of requested output values is approximately 20 to 30 CPU s.

The main output of the program is in the form of graphs. Two versions of the program have been developed. The first is with digital plots. The graphs are obtained in regular printers or CRT's together with the rest of the output. Only standard WRITE FORTRAN statements have been used for the digital plots. This version is faster, independent of computer equipment and software, and needs less in-core memory. A 64k-32bit CPU is enough for this version of the program. The second version is with continuous plots. This version requires a CALCOMP package for the plotting routines, and graphical output equipment. A version based on PLOT-10 software is under development.

Up to now the program has been implemented in CDC, VAX, CRAY, DEC-10, ND-100, ND-500, and UNIVAC. Versions for microcomputers are under development.

GENERAL REMARKS

The program is well documented. In addition to the user manual, refs. 1–6 explain in detail the theoretical aspects behind the computational and simulation methods as well as the concept of hydrodynamic forces and short-crested waves. Detailed examples5 show the correct use of the program. The accuracy of the program has been checked with in situ measured response values at the Hood-Canal floating bridge.1

The current versions of the program are for linear analysis. Nonlinearities can take place in the wave loading and the anchor cable behaviour. A version of the program taking care of these nonlinearities in the time domain analysis is under development.

Additional programs have been developed to take care of other aspects of the design of floating structures. These aspects are the evaluation of the hydrodynamic coefficients,6 the wave characteristics,4 the wave prediction from the wind in restricted fetch areas,5,10 and the handling and manipulation of time series in the frequency and time domain.6

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