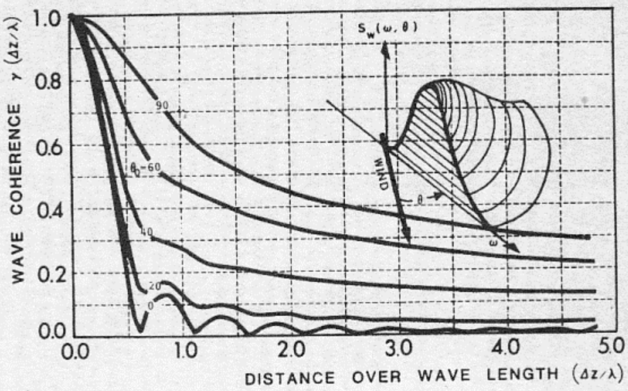
Wave Coherence Along Continuous Structures for Directional Spectra Models



C. Georgiadis, Ph.D.

CONTENTS

WAVE COHERENCE ALONG CONTINUOUS STRUCTURES FOR DIRECTIONAL SPECTRA MODELS

1. INTRODUCTION

An important factor in the dynamic analysis of floating structures, especially long ones, is the wave correlation along the structure. This wave correlation depends upon the wave energy distribution in the sea state. Once a mathematical model of the sea state is developed the longitudinal wave correlation can be calculated. The inverse problem is to determine an appropriate measure of the sea state from measurements of longitudinal correlation. For the mathematical model of the sea state, the concept of directional wave spectrum is used. Unfortunately there is not enough information in the literature to be able to tailor a directional wave model to a region and from that determine the wave correlation along the structure. Here a program for correlating wave data, wave correlation along a long structure and directional wave models is presented. Different models of wave energy spreading functions have been chosen, and a computer program WAVCOHR has been developed to compute the wave correlation and to fit an exponential approximation to this correlation. Some conclusions have been drawn for appropriate wave spreading functions, using wave measurements at certain regions.

2. DIRECTIONAL SPECTRA MODELS

The directional wave spectrum is assumed of the form:

$$S(f,\theta) = S_w(f).\Psi(\theta) \qquad (2.1)$$

where $S_w(f)$ is one of the unidirectional spectrum models:

a. Pierson-Moskowitz:

$$S_w(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left[-\frac{5}{4}\left(\frac{f}{fs}\right)^{-4}\right] \qquad (2.2.a)$$

b. JONSWAP:

$$S_w(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left[-\frac{5}{4}\left(\frac{f}{f_s}\right)^{-4}\right]\gamma^{\exp\left[-\frac{(f-f_s)^2}{2\sigma^2 f_s^2}\right]} \qquad (2.2.b)$$

and $\Psi(\theta)$ is a spreading function [1], [8], [10], giving a directional distribution of the wave energy. The following commonly used expressions for spreading functions have been incorporated in the computer program:

$$\Psi_1(\theta) = C\cos^s(\theta-\theta_0) \qquad (2.3.a)$$

$$\Psi_2(\theta) = C\cos^{2s}\left(\frac{\theta-\theta_0}{2}\right) \qquad (2.3.b)$$

$$\Psi_3(\theta) = C\exp\left[s\cos(\theta-\theta_0)\right] \qquad (2.3.c)$$

In the above expressions θ_0 represents the mean wave or wind direction , and C is a normalizing constant determined such that:

$$\int_{\theta_0-\pi/2}^{\theta_0+\pi/2} \Psi(\theta)\,d\theta = 1 \qquad (2.4)$$

3. WAVE COHERENCE ALONG A STRAIGHT LINE

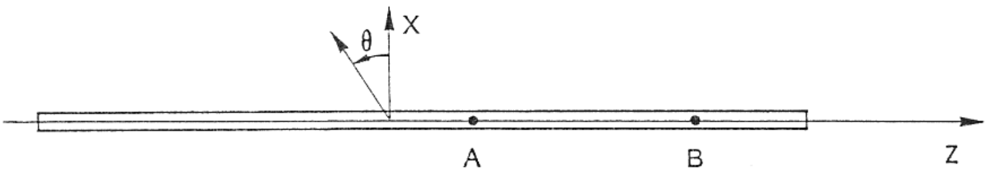


Figure 3.1 Geometrical Definitions

For a co-ordinate system of Figure 3.1 the sea state at a point (X,Z) can be represented:

$$\overset{2}{\eta}(X,Z,t) =$$

$$\int_{-\infty}^{\infty}\int_{-\frac{\pi}{2}+\theta_0}^{\frac{\pi}{2}+\theta_0} S_w(\omega,\theta)\, e^{i[kX\cos\theta + kZ\sin\theta - \omega t + \phi(\omega,\theta)]}\, d\theta\, d\omega \tag{3.1}$$

where $\phi(\omega,\theta)$ is a random angle between 0 and 2π, and θ_0 is the mean wind direction. The cross-spectral density between $\eta(X_A,Z_A,t)$ and $\eta(X_B,Z_B,t)$ will be:

$$S_{w(AB)}(\omega) = \int_{-\frac{\pi}{2}+\theta_0}^{\frac{\pi}{2}+\theta_0} S_w(\omega,\theta)\, e^{i[k(X_A-X_B)\cos\theta + k(Z_A-Z_B)\sin\theta]}\, d\theta \tag{3.2}$$

and for points on the bridge axis (X=0), the above relation becomes:

$$S_{w(AB)}(\omega) = \int_{-\frac{\pi}{2}+\theta_0}^{\frac{\pi}{2}+\theta_0} S_w(\omega,\theta)\, e^{ik(Z_A-Z_B)\sin\theta}\, d\theta =$$

$$= \int_{-\frac{\pi}{2}+\theta_0}^{\frac{\pi}{2}+\theta_0} S_w(\omega,\theta)\, e^{i2\pi(\frac{\Delta Z}{\lambda})\sin\theta}\, d\theta \tag{3.3}$$

The coherence function for the wave field between points A and B is:

$$\gamma_{w(AB)} = \frac{|S_{w(AB)}(\omega)|}{\sqrt{S_{w(AA)}(\omega)S_{w(BB)}(\omega)}} = \frac{|S_{w(AB)}(\omega)|}{S_{w(\omega)}} \tag{3.4}$$

where $S_{w(AA)}(\omega) = S_{w(BB)}(\omega) = S_w(\omega)$ has been assumed and $\omega = 2\pi f$.

Representing the wave spectrum by Eq. 2.1 we get the coherence between two points at distance ΔZ on the bridge:

$$\gamma\left(\frac{\Delta Z}{\lambda}\right) =$$

$$= \sqrt{\left[\int_{-\frac{\pi}{2}+\theta_0}^{\frac{\pi}{2}+\theta_0} \Psi(\theta)\cos\left(2\pi\left(\frac{\Delta Z}{\lambda}\right)\sin\theta\right) d\theta\right]^2 + \left[\int_{-\frac{\pi}{2}+\theta_0}^{\frac{\pi}{2}+\theta_0} \Psi(\theta)\sin\left(2\pi\left(\frac{\Delta Z}{\lambda}\right)\sin\theta\right) d\theta\right]^2} \tag{3.5}$$

Using numerical integration and integrating (3.5) for the spreading functions (2.3.a, b, c) Figures 3.2, 3.3, 3.4 are obtained. For some applications [4,7] the wave coherence can be approximated with (3.6).

$$\gamma_w\left(\frac{\Delta Z}{\lambda}\right) = e^{-\alpha\left(\frac{\Delta Z}{\lambda}\right)^{\beta}} \tag{3.6}$$

Curves of this form are shown in Fig. 3.5.

A least square fitting of curves of the above type to the curves of Eq. 3.5 give results for α and β coefficients shown in Tables 3.1, 3.2, 3.3 for the different kinds of spreading functions.

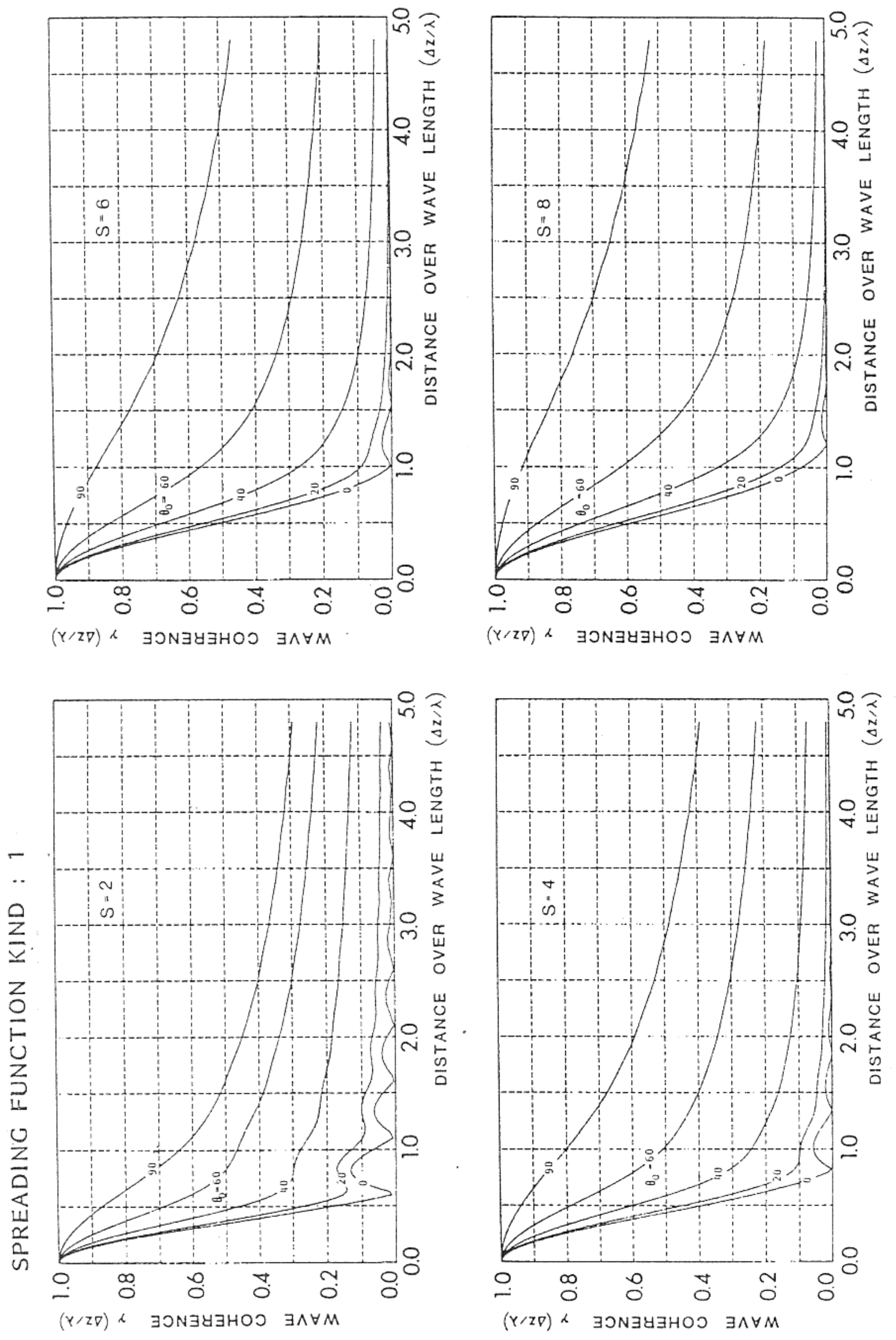


Figure 3.2.a Wave cohorence $\gamma(\Delta Z/\lambda)$ for spreading function of the form $\Psi_1(\theta) = C\cos^S(\theta-\theta_0)$ $(\theta_0 = 0^o, 20^o, 40^o, 60^o, 90^o)$

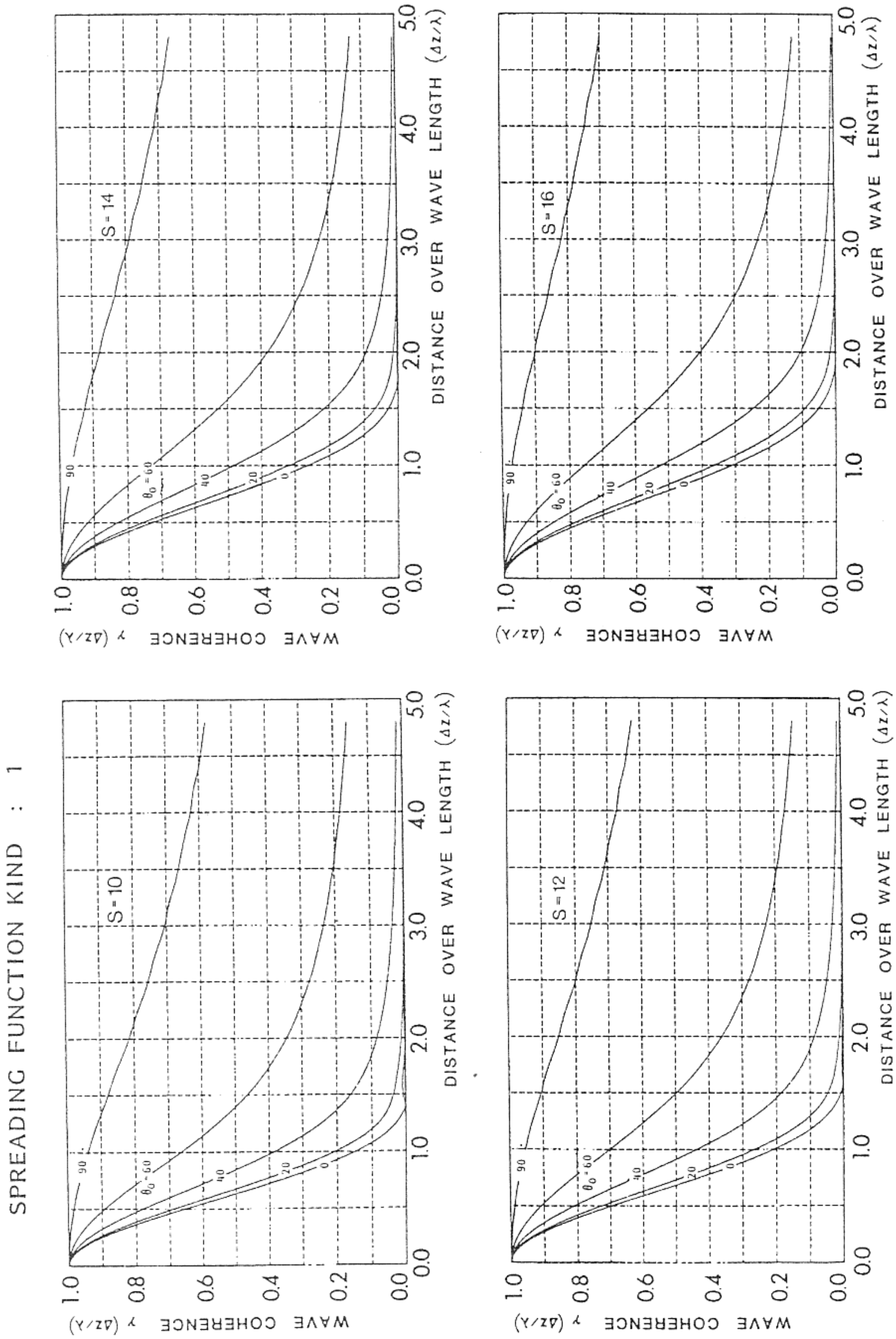


Figure 3.2.b Wave cohorence $\gamma(\Delta Z/\lambda)$ for spreading function of the form $\Psi_1(\theta) = C\cos^s(\theta-\theta_0)$ ($\theta_0 = 0^o, 20^o, 40^o, 60^o, 90^o$)

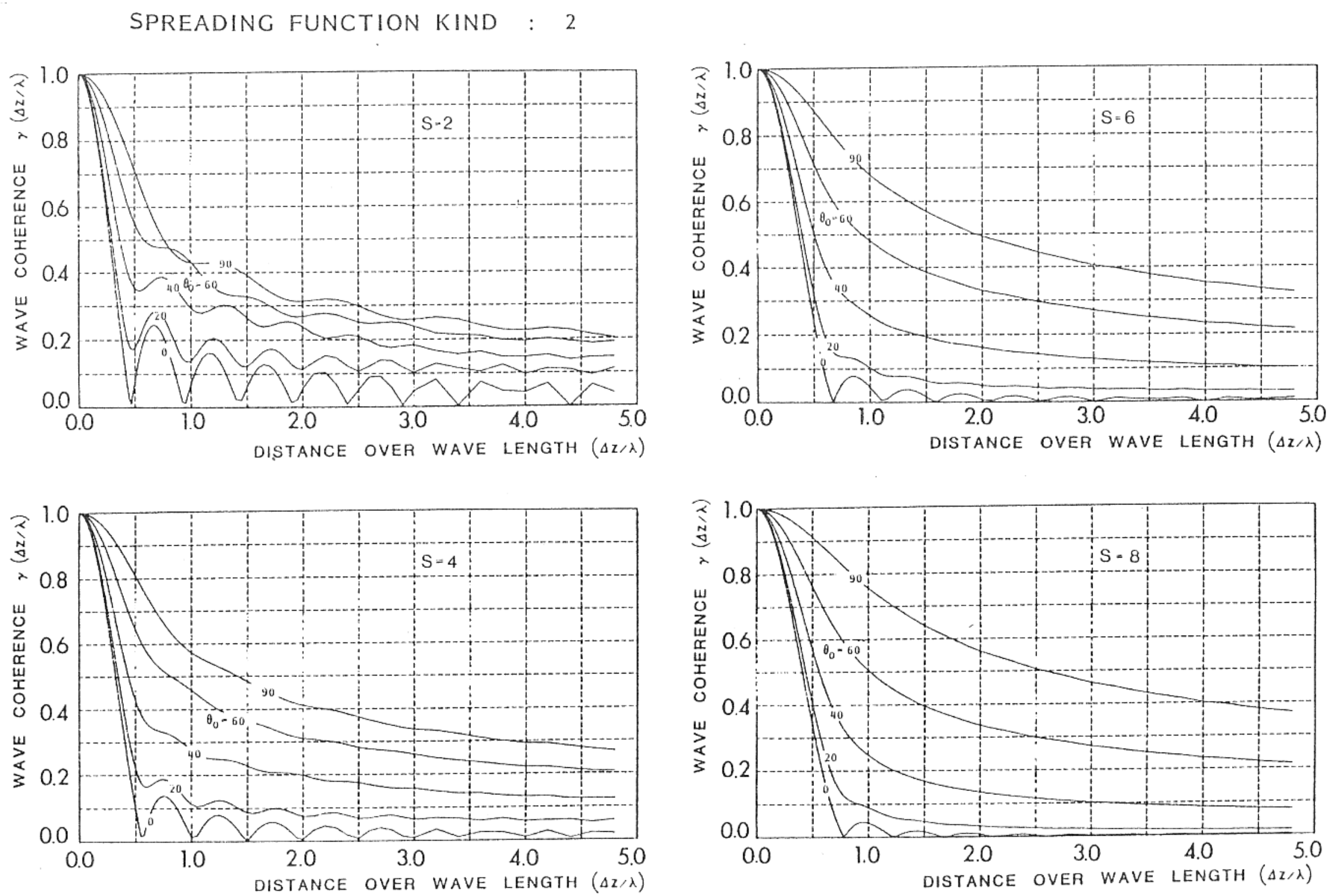


Figure 3.3.a Wave coherence $\gamma(\Delta Z/\lambda)$ for spreading function of the form $\Psi_2(\theta) = C\cos^{2s}\left(\frac{\theta-\theta_0}{2}\right)$ ($\theta_0 = 0^o, 20^o, 40^o, 60^o, 90^o$)

7

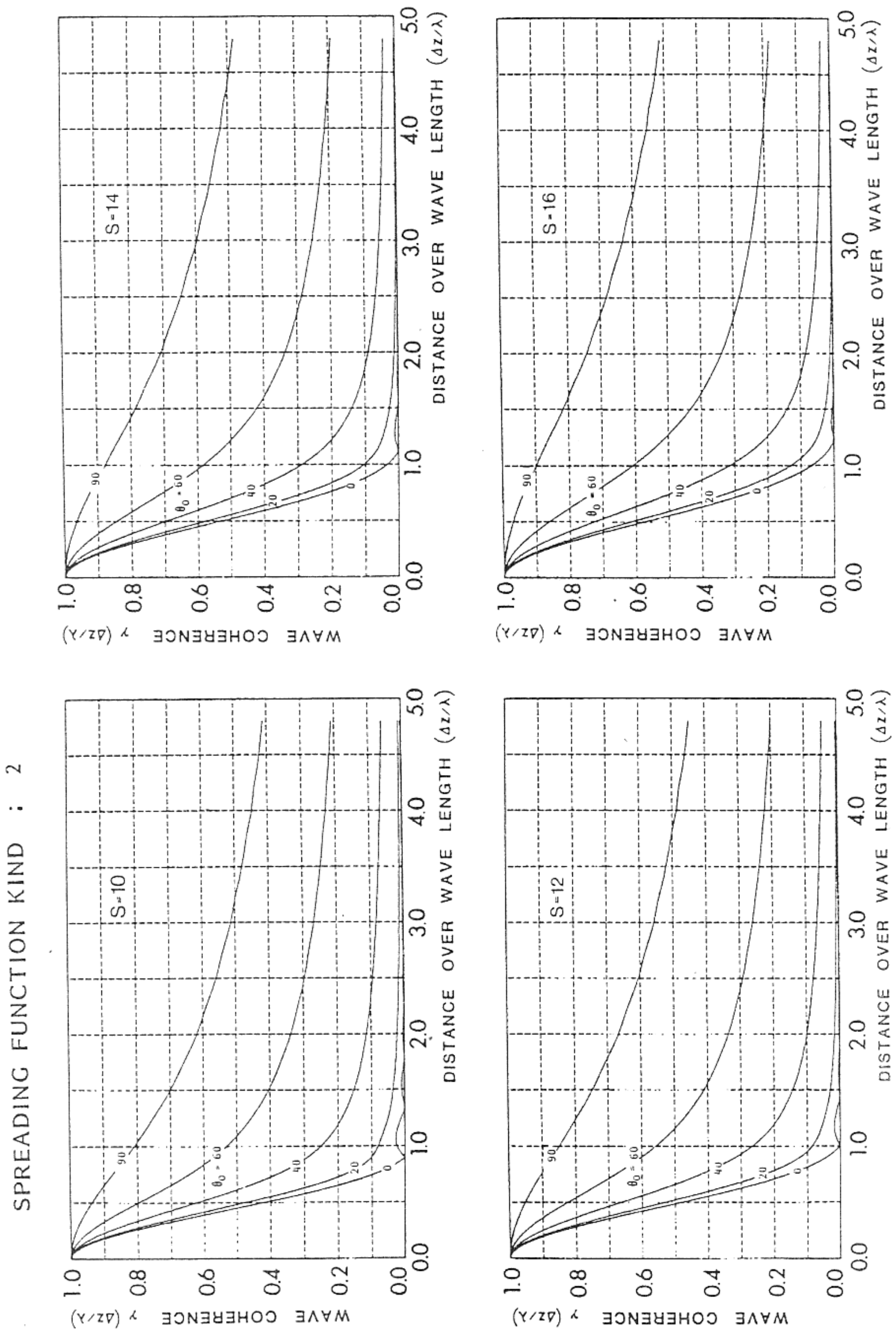


Figure 3.3.b Wave cohorence $\gamma(\Delta Z/\lambda)$ for spreading function of the form
$\Psi_2(\theta) = C \cos^{2s}\left(\frac{\theta-\theta_0}{2}\right)$ ($\theta_0 = 0^o, 20^o, 40^o, 60^o, 90^o$)

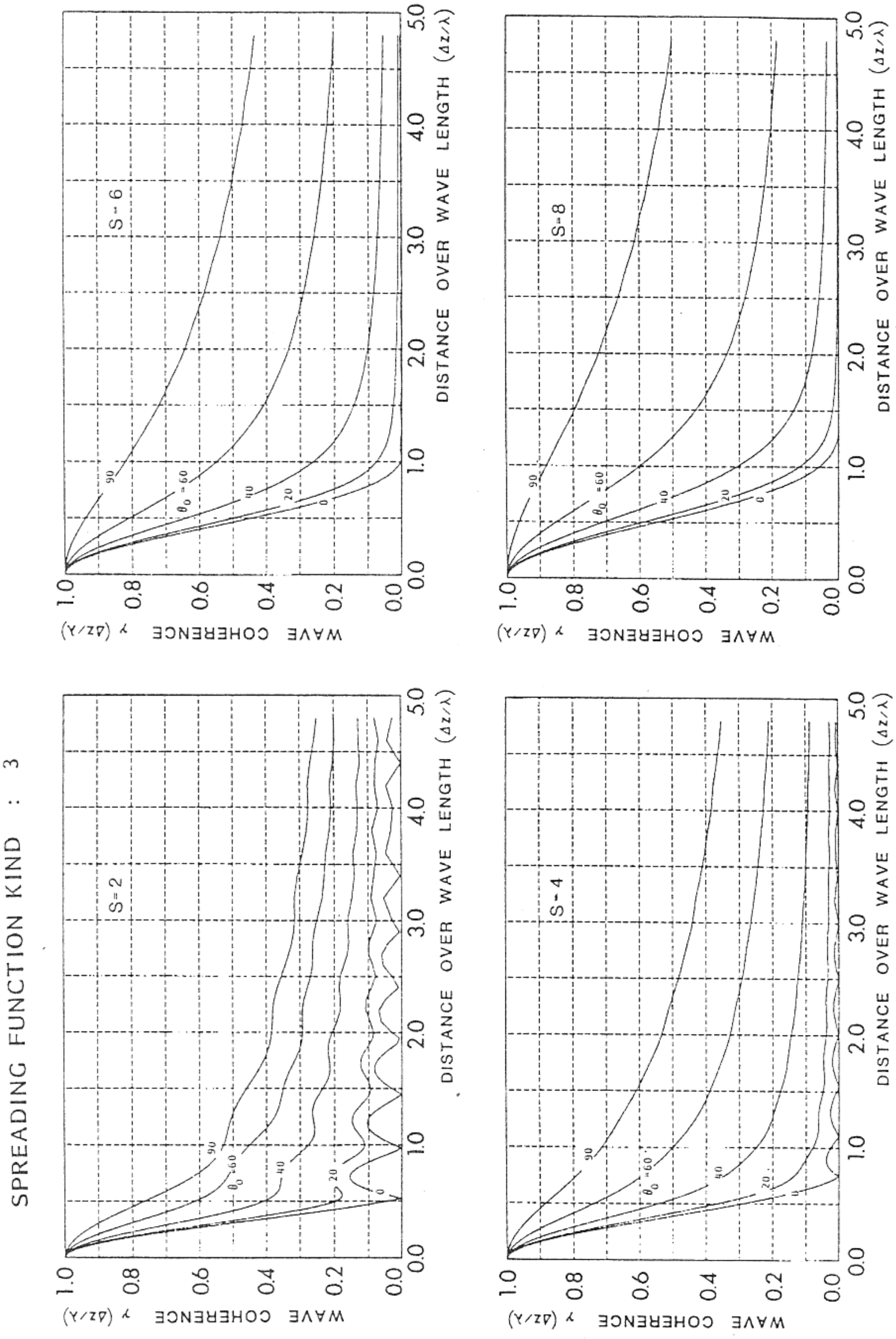


Figure 3.4.a Wave cohorence $\gamma(\Delta Z/\lambda)$ for spreading function of the form
$\Psi_3(\theta) = C \exp[s \cos(\theta - \theta_0)]$ ($\theta_0 = 0^o, 20^o, 40^o, 60^o, 90^o$)

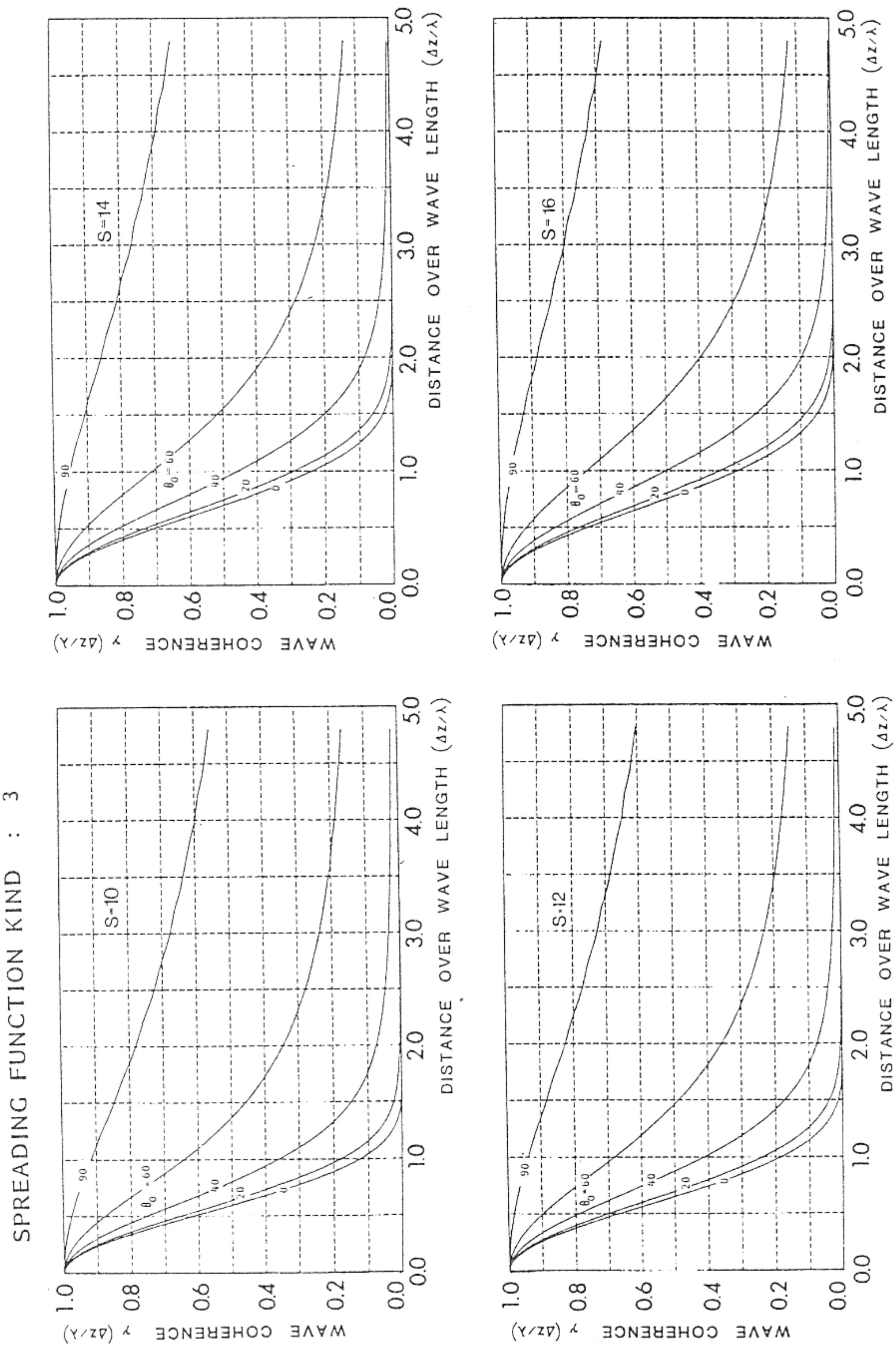


Figure 3.4.b Wave cohorence $\gamma(\Delta Z/\lambda)$ for spreading function of the form $\Psi_3(\theta) = C \exp[s \cos(\theta-\theta_0)]$ ($\theta_0 = 0^o, 20^o, 40^o, 60^o, 90^o$)

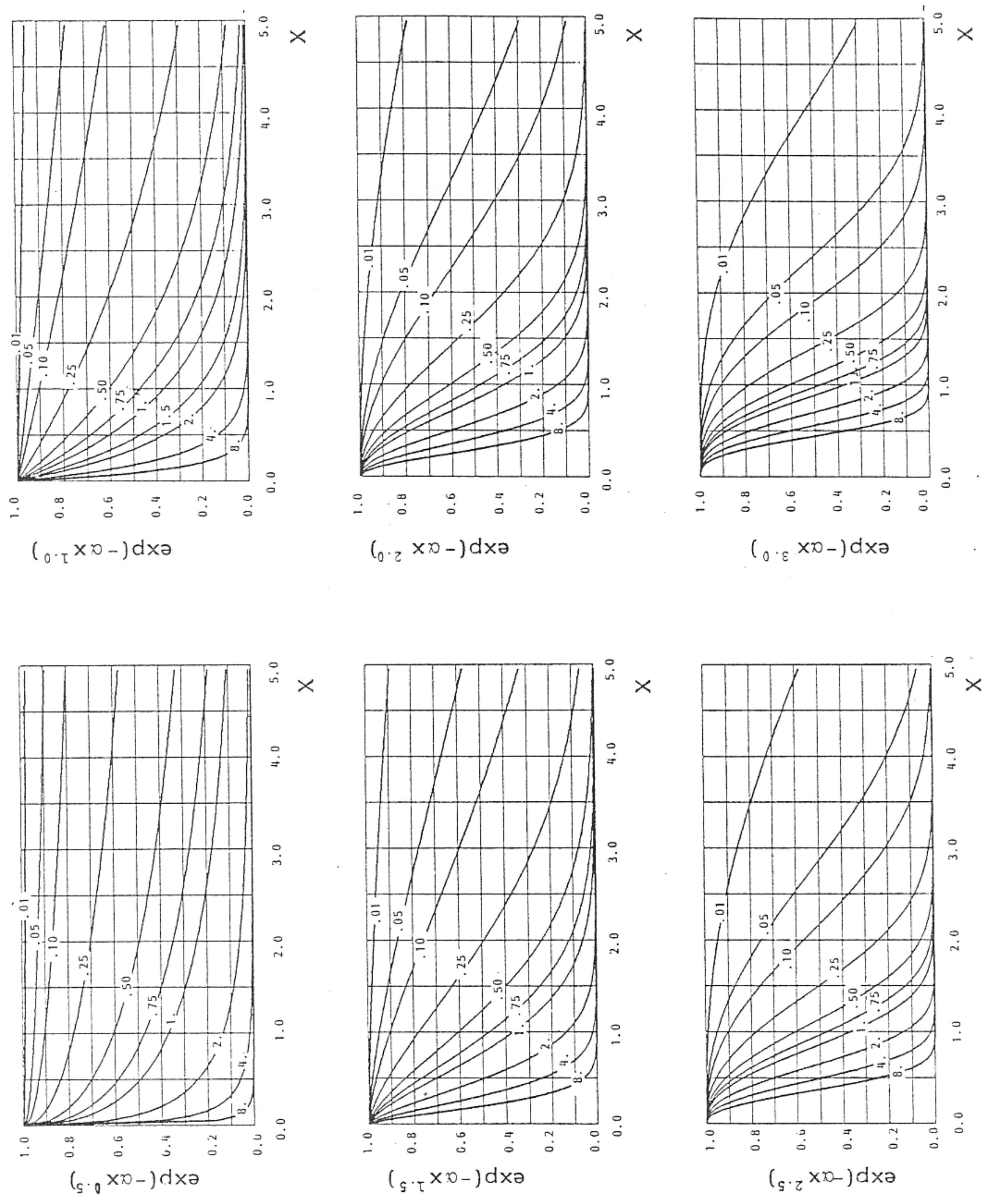


Figure 3.5 Curves of the form $\exp(-\alpha x^{\beta})$

β = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0

α = 0.01, 0.05, 0.10, 0.25, 0.50, 0.75, 1.0, 2.0, 4.0, 8.0

(α on curves)

TABLE 3.1

$\theta_0=$		0^0	20^0	40^0	60^0	90^0
s= 2.00	α	4.41	1.42	.90	.53	.29
	β	1.89	1.22	1.18	1.21	1.44
s= 4.00	α	4.19	1.49	.87	.44	.17
	β	2.08	1.48	1.35	1.37	1.57
s= 6.00	α	2.96	1.68	.82	.38	.11
	β	2.06	1.77	1.50	1.49	1.65
s= 8.00	α	2.30	1.60	.76	.34	.07
	β	2.07	1.91	1.61	1.57	1.71
s=10.00	α	1.85	1.44	.70	.31	.05
	β	2.05	1.96	1.69	1.64	1.76
s=12.00	α	1.56	1.27	.64	.28	.04
	β	2.04	1.99	1.74	1.70	1.79
s=14.00	α	1.35	1.13	.62	.26	.03
	β	2.03	2.00	1.82	1.74	1.81
s=16.00	α	1.19	1.01	.57	.24	.03
	β	2.03	2.00	1.87	1.78	1.82

Table 3.1. Results for α, β coefficients fitting exponential curves of the form (3.6) to curves of fig 3.2 spreading function $\Psi_1(\theta) = C\cos^s(\theta-\theta_0)$

TABLE 3.2

	$\theta_0=$	0^0	20^0	40^0	60^0	90^0
s= 2.00	α	2.33	1.29	.93	.68	.53
	β	1.27	.97	1.00	1.13	1.27
s= 4.00	α	3.73	1.40	.92	.58	.37
	β	1.71	1.13	1.12	1.15	1.34
s= 6.00	α	5.96	1.45	.90	.51	.27
	β	2.09	1.27	1.24	1.23	1.39
s= 8.00	α	4.63	1.47	.89	.46	.20
	β	2.07	1.39	1.29	1.32	1.47
s=10.00	α	3.78	1.56	.87	.43	.16
	β	2.06	1.55	1.38	1.39	1.52
s=12.00	α	3.26	1.78	.84	.40	.13
	β	2.06	1.75	1.46	1.45	1.58
s=14.00	α	2.81	1.81	.81	.37	.10
	β	2.07	1.86	1.52	1.50	1.61
s=16.00	α	2.45	1.75	.78	.35	.09
	β	2.06	1.92	1.58	1.53	1.66

Table 3.2. Results for α, β coefficients fitting exponential curves of the form (3.6) to curves of Fig 3.3.

Spreading function $\Psi_2(\theta) = C\cos^{2s}(\frac{\theta-\theta_0}{2})$

TABLE 3.3

	$\theta_0 =$	0^0	20^0	40^0	60^0	90^0
s= 2.00	α	9.02	1.37	.93	.62	.43
	β	2.12	1.07	1.08	1.11	1.28
s= 4.00	α	5.34	1.47	.90	.49	.25
	β	2.09	1.32	1.29	1.27	1.38
s= 6.00	α	3.50	1.60	.86	.42	.15
	β	2.05	1.61	1.41	1.40	1.49
s= 8.00	α	2.59	1.95	.80	.37	.10
	β	2.05	1.95	1.55	1.51	1.58
s=10.00	α	2.04	1.67	.75	.33	.07
	β	2.03	1.98	1.64	1.58	1.65
s=12.00	α	1.68	1.41	.69	.30	.05
	β	2.02	1.99	1.72	1.65	1.71
s=14.00	α	1.4	1.22	.66	.27	.04
	β	2.02	1.99	1.81	1.70	1.74
s=16.00	α	1.25	1.07	.62	.25	.03
	β	2.01	1.99	1.87	1.75	1.78

Table 3.3. Results for α, β coefficients fitting exponential curves of the form (3.6) to the curves of Fig. 3.4. Spreading function $\Psi_3(\theta) = C\exp\,[s \cos(\theta-\theta_0)]$

5. DISCUSSION AND CONCLUSION

The wave coherence in a convenient form for comparison with measured coherence along a structure or array of wave gauges is readily obtained from the program WAVCOHR and/or the results in Figures 3.2 - 3.4. These results may also be used for obtaining the wave coherence in terms of a defined sea state. For use in the dynamic analysis of structures the further approximation by Eq. (3.6) may be convenient [4]. For curved structures or wave gauge arrays not in a straight line WAVCOHR and the results presented here may still be used by taking account of the changing direction of the normal to the structure or array relative to the wind direction. The curves for $\theta = 90^0$ in Figures 3.2 - 3.4 give the coherence in the direction of the wind.

Relatively little field data is available for correlation with the directional spectra models. Measurements on the Hood Canal Floating Bridge - [5, 6, 7, 9] indicate that for the short-crested waves in the relatively short fetch of the Hood Canal Basin the wave correlation goes to zero in something less than about 0.6λ corresponding to spreading function models with values of s may be even less than the lowest value of 2.0 used in the Figures here. In particular these measurements indicate a rapid drop off of coherency for all wind directions in contrast to all of the models included in the report which show measurable coherence for other than zero wind direction. Such measured coherences can be approximated by the exponential functions Fig. 3.5 but is not well represented by the spreading function models.

REFERENCES

1. Borgman,L.E.: *The Estimation of Parameters in a Circular Normal Two-Dimensional Wave Spectrum*, Tech. Report HEL 1-9, Hydraulic Engineering Laboratory, University of California, Berkely, 1967.

2. Borgman,L.E.: *Ocean Wave Simulation for Engineering Design*, J. ASCE WW4, Nov 1969 pp 557-583.

3. Borgman,L.E.: *Statistical Models for Ocean Waves and Wave Forces*, Advances in Hydroscience vol. 8, Academic Press 1972, London.

4. Georgiadis,C.: *Wave Induced Vibrations of Continuous Floating Structures*, Ph.D. dissertation, University of Washington, Seattle, May 1981.

5. Hartz,B.J.: *Summary Report on Structural Behaviour of Floating Bridges*, June 1972, Dept. of Civil Engineers, University of Washington, Report for Washington State Department of Highways, Contract Y-909, 2 volumes.

6. Hartz,B.J.: *Dynamic Response of the Hood Canal Floating Bridge*, Proceeding Second ASCE/EMD Specialty Conference on Dynamic Response of Structures, Atlanta, GA, Jan 15-16, 1981.

7. Hartz,B.J. and Georgiadis,C.: *A Finite Element Program for the Dynamic Analysis of Continuous Floating Structures in Short Crested Waves*, to be presented at the International Conference on Finite Element Methods, Shanghai, 2-6 August, 1982.

8. Kinsman,B.: *Wind Waves, their Gerevation and Propagation on the Ocean Surface*, Prentice Hall, 1965.

9. Mukherji,B.: *Dynamic Behaviour of a Continuous Floating Bridge*, Ph.D. dissertation, University of Washington, Seattle, 1972

10. Panicker,N.N.: *Review of Techniques for Directional Spectra*, Proc. Int. Symp. on Ocean Wave Measurement and Analysis, ASCE, vol. 1, 1974.

11. Sigbjörnsson, R.: *Stochastic theory of wave loading processes*, Journ. Eng. Struct. Vol. 1, Jan., 1979.

APPENDIX A

Least square fit of $y(x) = e^{-\alpha x^{\beta}}$ for coherence curves $\gamma(x)$ of Eq. (3.5).

From $y(x) = e^{-\alpha x^{\beta}}$ (B.1)

we get:

$$\ln(y) = -\alpha x^{\beta} \tag{B.2}$$

and

$$\ln[-\ln(y)] = \ln\alpha + \beta \ln x \tag{B:3}$$

The above relation represents a straight line and from least square fit formulas we have:

$$\beta = \frac{n \sum_{i=1}^{n} (\ln x_i).(\ln(-\ln y_i)) - \sum_{i=1}^{n} (\ln x_i).\sum_{i=1}^{n} (\ln(-\ln y_i))}{n(n-1)V^2} \tag{B.4}$$

$$\alpha = \bar{y} - b\bar{x} \tag{B.5}$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} (\ln(-\ln y_i)) \tag{B.6}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} (\ln x_i) \tag{B.7}$$

$$V^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (\ln x_i - \bar{x})^2 \tag{B.8}$$

For more accurate results of α, β the exponentional curves have been fitted to the part of the coherence curves at distances before the "ripples" appear which depends on the accuracy of the numerical integration of Eq. (3.5).